ТЕХНИЧЕСКИЕ НАУКИ | TECHNICAL SCIENCES

ИНФОРМАТИКА, ВЫЧИСЛИТЕЛЬНАЯ ТЕХНИКА И УПРАВЛЕНИЕ INFORMATICS, COMPUTER ENGINEERING AND MANAGEMENT

Современная наука и инновации. 2025. № 4. С. 10-21
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Modern Science and Innovations. 2025;(4):10-21
TECHNICAL SCIENCE
INFORMATICS, COMPUTER ENGINEERING AND MANAGEMENT

Научная статья
УДК 621.00.01
https://doi.org/10.37493/2307-910X.2025.4.1

Математическое моделирование акустических колебаний химически реагирующей двухфазной смеси монодисперсных твердых частиц в газообразном окислителе

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Аннотация. ***Введение.*** Явление самовозбуждения акустических колебаний за счёт теплообмена известно давно, но его изучение началось недавно из-за развития высокофорсированных камер сгорания. Эти колебания могут дестабилизировать горение, но также могут быть полезны, увеличивая тепловую нагрузку и сокращая время сгорания. Исследование волновых режимов горения важно, как теоретически, так и практически. В большинстве случаев изучали гомогенные среды, но реальные системы, такие как «жидкие капли – окислитель» или «твёрдые частицы – окислитель», демонстрируют уникальную волновую динамику. Для понимания этих процессов и управления колебаниями необходимо более глубокое изучение. ***Цель.*** На основе модели взаимодействующих континуумов рассмотрена задача о слабонелинейных волновых возмущениях в ограниченном объёме химически реагирующей двухфазной смеси монодисперсных твердых частиц в газообразном окислителе. Исследование учитывает, что динамическое и тепловое взаимодействие фаз влияет на диссипацию и дисперсию фазовой скорости звука. Метод медленно меняющихся амплитуд позволил свести систему уравнений сохранения массы, энергии и импульса к нелинейному волновому уравнению. Получены уравнения для установившихся амплитуд колебаний. Обсуждено влияние дисперсии, вызванной различием температур и скоростей фаз, на нелинейное взаимодействие стоячих волн. Показано, что зависимость скорости звука от частоты ограничивает перекачку энергии вверх по спектру, увеличивая амплитуды первых обертонов. ***Материалы и методы.*** С помощью метода медленно меняющихся амплитуд система уравнений сохранения массы, энергии и импульса для обеих фаз сведена к единственному нелинейному волновому уравнению. ***Результаты и обсуждение.*** Получены уравнения для определения значений установившихся амплитуд колебаний. Исследовано влияние дисперсии, вызванной несовпадением температур и скоростей фаз газовзвеси, на нелинейное взаимодействие стоячих волн. ***Заключение.*** В данной статье представлено исследование поведения акустических возмущений в ограниченном объёме горящей газовзвеси.

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Целью исследования было получение единственного нелинейного волнового уравнения, описывающего эволюцию давления в топке. В основу анализа положено предположение о малости влияния нелинейности, дисперсии и неконсервативности колебаний на амплитуды волн. Это позволило применить метод разложения решения по собственным модам линейной консервативной задачи для решения полученного уравнения. Процедура разложения позволила свести исходное волновое уравнение к бесконечной системе обыкновенных дифференциальных уравнений для комплексных амплитуд. В рамках этого подхода были найдены значения установившихся амплитуд стоячих волн, что представляет собой важный вклад в понимание динамических процессов в системах горения. Таким образом, исследование демонстрирует высокую степень аналитической строгости и математической точности, а также глубокое понимание фундаментальных принципов акустики и горения. Полученные результаты могут быть использованы для дальнейшего развития теоретических моделей и экспериментальных исследований в области горения и акустических процессов.

Ключевые слова: вибрационное горение, акустические колебания, волновые режимы, химически реагирующая газовзвесь

Для цитирования: Янукян Э. Г. Математическое моделирование акустических колебаний химически реагирующей двухфазной смеси монодисперсных твердых частиц в газообразном окислителе // Современная наука и инновации. 2025. № 4. С. 10-21. https://doi.org/10.37493/2307-910X.2025.4.1

Конфликт интересов: Авторы заявляет об отсутствии конфликта интересов.

Статья поступила в редакцию 01.10.2025;
одобрена после рецензирования 01.11.2025;
принята к публикации 01.12.2025.

Research article

Mathematical modeling of acoustic oscillations of chemically reacting two-phase mixture of monodisperse solid particles in a gaseous oxidant

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Abstract. ***Introduction.*** The phenomenon of self-excitation of acoustic oscillations due to heat transfer has been known for a long time, but its study has only recently begun due to the development of high-pressure combustion chambers. These oscillations can destabilize combustion, but they can also be beneficial by increasing the heat load and reducing the combustion time. The study of wave modes in combustion is important both theoretically and practically. Most studies have focused on homogeneous media, but real systems such as "liquid droplets-oxidizer" or "solid particles-oxidizer" exhibit unique wave dynamics. A deeper understanding is needed to comprehend these processes and manage fluctuations. ***Goal.*** Based on the model of interacting continua, the problem of weakly nonlinear wave disturbances in a limited volume of a chemically reacting two-phase mixture of monodisperse solid particles in a gaseous oxidant is considered. The study takes into account that the dynamic and thermal interaction of the phases affects the dissipation and dispersion of the phase sound velocity. The method of slowly changing amplitudes allowed the system of mass, energy, and momentum conservation equations to be reduced to a nonlinear wave equation. Equations for the steady-state amplitudes of oscillations were obtained. The effect of dispersion caused by the difference in temperatures and phase velocities on the nonlinear interaction of standing waves is discussed. It is shown that the dependence of the speed of sound on frequency limits the transfer of energy up the spectrum, increasing the amplitudes of the first overtones. ***Materials and methods.*** Using the method of slowly varying amplitudes, the system of mass, energy, and momentum conservation equations for both phases is reduced to a single nonlinear wave equation. ***Results and discussion.*** Equations are obtained for determining the values of the established amplitudes of oscillations. The influence of dispersion caused by the non-coincidence of temperatures and velocities of gas-suspension phases on the nonlinear interaction of standing waves is investigated. ***Conclusion.*** This article presents a study of the behavior of acoustic disturbances in a limited volume of burning gas suspension. The goal of the study was to obtain a single nonlinear wave equation that describes the evolution of pressure in the furnace. The analysis is based on the assumption that the effects of nonlinearity, dispersion, and non-conservation of oscillations on wave amplitudes are negligible. This assumption allows us to use the method of decomposition of the solution into eigenmodes of the linear conservative problem to solve the obtained equation. The decomposition procedure reduces the original wave equation to an infinite system of ordinary differential equations for complex amplitudes. Within this approach, the values of the steady-state amplitudes of the standing waves were found, which represents an important contribution to the understanding of the dynamic processes in combustion systems. Thus, the study demonstrates a high degree of analytical rigor and mathematical accuracy, as well as a deep understanding of the fundamental principles.

Key words: vibration combustion, acoustic oscillations, wave modes, chemically reacting gas suspension
For citation: Yanukyan EG. Mathematical modeling of acoustic oscillations of chemically reacting two-phase mixture of monodisperse solid particles in a gaseous oxidant. *Modern Science and Innovations*. 2025;(4):10-21. (In Russ.). https://doi.org/10.37493/2307-910X.2025.4.1

Conflict of interest: the authors declare no conflicts of interests.

The article was submitted 01.10.2025;
approved after reviewing 01.11.2025;
accepted for publication 01.12.2025.

Introduction. The effect of excitation and amplification of acoustic vibrations due to heat removal has been known for a long time (for example, the phenomenon of "singing flames" and the Rijke tube [1], in which heat is supplied to the air using a hot grid). However, interest in the self-excitation of sound waves by means of combustion arose relatively recently in connection with the creation of highly accelerated furnaces and combustion chambers of rocket engines, in which strong pressure fluctuations were discovered [1-5]. These vibrations can significantly disrupt the combustion process and lead to the destruction of the structural elements of the furnace or engine. On the other hand, it often turns out to be advantageous to maintain an oscillatory combustion mode [6-8], since this increases the thermal stress of the furnaces, significantly intensifies the processes of heat and mass transfer and, as a result, reduces the combustion time of the fuel. In addition, the transition to vibrational combustion opens up great prospects in metallurgy, the chemical industry, etc. [1]. Thus, the problem of wave modes of behavior of reacting systems is of not only theoretical but also practical interest. Basically problem of the theory of vibrational combustion was considered in relation to homogeneous media (with the exception of [9, 10]). In real systems, burning systems are mixtures of the "liquid droplets - oxidizer" or "solid particles - oxidizer" type, whose wave dynamics differ from those in homogeneous media. For example, heat and mass transfer processes lead to the dissipation of sound wave energy in inert systems and can amplify waves in reacting two-phase systems.

In this paper, the problem of weakly nonlinear wave disturbances in a confined volume of a chemically reacting two-phase mixture of monodisperse solid particles in a gaseous oxidizer is considered using a model of interacting interpenetrating continua. The study corresponds to a situation where the dynamic and thermal interaction between the phases determines not only the dissipation of sound wave energy but also the dispersion of the phase velocity of sound. Using the method of slowly varying amplitudes, the system of equations for the conservation of mass, energy, and momentum for both phases is reduced to a single nonlinear wave equation. Equations are derived for determining the values of steady-state oscillation amplitudes. The influence of dispersion caused by the mismatch of temperatures and phase velocities of the gas suspension on the nonlinear interaction of standing waves is discussed in detail. It is shown that the dependence of the speed of sound on frequency leads to a limitation of energy transfer up the spectrum and, thereby, to an increase in the amplitudes of the first overtones.

Materials and methods of the study. We will consider the acoustic vibrations of a mixture consisting of solid particles reacting in a kinetic regime, suspended in a gaseous oxidizer. For simplicity, we will assume that the particles have identical thermophysical properties. Assuming that there are a sufficient number of particles at distances of the order of the wavelength, we will describe the oscillation processes in the system using methods of continuum mechanics. We will assume that the chemical reaction occurs in a purely heterogeneous regime without changing the molar content, for example $C + O_2 \to CO_2$. It is known that the combustion reaction of carbon is significantly more complex [11]. However, taking into account intermediate reactions would significantly complicate the analysis and lead to more complex calculations, but at the same time would not significantly affect the effects considered here. Neglecting the dissipation of sound waves due to viscosity and thermal conductivity in the gas volume, we write the system of equations of motion of a two-phase mixture in the form:

$$\frac{\partial(\varepsilon d_0)}{\partial t} + \frac{\partial(\varepsilon d_0 u)}{\partial x} = -I\,;\ \frac{\partial(\rho d_1)}{\partial t} + \frac{\partial(\rho d_1 v)}{\partial x} = I\,;$$

$$\frac{\partial(\varepsilon n_0)}{\partial t} + \frac{\partial(\varepsilon n_0 u)}{\partial x} = -I_0\,;\ \rho = 1 - \varepsilon\,,\ d_0 = n_0 + n_1\,;$$

$$\varepsilon d_0\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)u = -\frac{\partial p}{\partial x} - f + I(u - v)\,;$$

$$\rho d_1\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)v = f\,;\ p = R_0 n_0 T + R_1 n_1 T_0\,;\ (1)$$

$$\varepsilon d_0 c_v\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)T_0 + p\frac{\partial}{\partial x}(\varepsilon u + \rho v) =$$

$$= Q_0 - \alpha(T_0 - T_e) - \frac{p}{d_0} I\left(1 - \frac{d_0}{d_1}\right) + f(u - v) - I\frac{(v-u)^2}{2}\,;$$

$$I = -(g-1)I_0\,;\ \rho d_1 c_1\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)T_1 = -Q_1\,;\ Q_0 - Q_1 = L I_0\,.$$

Here p, T_0, is the average pressure and temperature of the mixture of the oxidizer and reaction product, T_e is the ambient temperature, α is the effective heat transfer coefficient, d_0, d_1 and u, v are the densities and average velocities of the carrier gas and particles, respectively, ρ is the volume concentration of the particles, n_0 and n_1 are the densities of the oxidizer and reaction product, T_1 is the average temperature of the particles, L is the heat of reaction, g is the ratio of the molecular weights of the reaction product and the oxidizer, c_v is the specific heat capacity of the gas at constant volume, c_1 is the specific heat capacity of the material of the particles.

A stationary state, in which the suspension is considered motionless $(u = v = 0)$, is achieved when the heat release due to the chemical reaction is equal to the heat removal through the channel walls, $LI_0 = \alpha(T_0 - T_e)$ i.e.

When analyzing the wave motions of a two-phase reacting mixture, the system of equations (1) can be closed by the following expressions for the heat fluxes Q_0, Q_1, the oxidizer flux I_0, and the interphase interaction force f :

$$f = \frac{\rho d_1}{\tau_d}(u - v)\,;\, \tau_d = \frac{2a^2 d_1}{9\nu_0 d_0}$$

$$Q_0' = \frac{3\rho\lambda_0}{a^2}\left(T_1' - T_0'\right);\; \frac{dT_1'}{dt} = \frac{T_0' - \left(1 - p_*\right)T_1'}{\tau_t} \quad (2)$$

$$I_0' = \frac{3\rho\lambda_0 p_*}{a^2 L}T_1';\; p_* = \frac{E}{RT_{01}}\left(T_{01} - T_0\right);\; \tau_t = \frac{a^2 d_1 c_1}{3\lambda_0},$$

where the prime denotes the quantities disturbed by the wave, λ_0 is the thermal conductivity coefficient of the carrier gas, T_{01} is the stationary temperature distribution in the vicinity of the particle, E is the activation energy, R is the gas constant, T_0', T_1' is the disturbance of the temperature in the gas and inside the particle, respectively, by the sound wave, a is the particle radius, ν_0 is the kinematic viscosity of the mixture of the oxidizer and the reaction product, τ_d is the relaxation time of the phase velocities in the Stokes flow around the particle. Expressions (2) were obtained for a highly dilute gas suspension ($\rho << 1$) for the case of quasi-stationary flow around the particles. The latter is valid in a situation where the wave frequency satisfies the inequality

$$\tau_d'\omega \sim \tau_t'\omega << 1,\; \tau_d' = \frac{a^2}{\nu_0},\; \tau_t' = \frac{a^2}{\chi_0},$$

where τ_d' and τ_t' - relaxation times for the processes of establishing a quasi-stationary distribution of velocity and temperature in the vicinity of a particle, χ_0 - thermal diffusivity of the gas phase.

We will further assume that the oxidizer density n_0 does not change over a time of the order of the wave period, i.e. $\tau_n\omega >> 1$. Here $\tau_n = \frac{a^2(1+x)}{3\rho D x}$, , and is x a parameter characterizing the ratio of the rate of chemical reaction to the rate of oxidizer supply to the particle surface through diffusion. For the kinetic combustion regime

$$x = \frac{az\exp\left(-\frac{E}{RT_0}\right)}{D}$$

much less than one, where z is the pre-exponential factor, D is the diffusion coefficient.

Research results and discussion. The behavior of acoustic vibrations in an inert or chemically reacting gas suspension depends on the processes of dynamic and thermal interaction between the phases. These phenomena cause dissipation of wave energy and dispersion of its phase velocity. Note that the influence of heat and momentum exchange has the strongest impact on the propagation of acoustic waves under the condition $\tau_d\omega \sim \tau_t\omega \sim 1$.

Let us assume that the times of heating of the gas $\tau_к$ by the heat of chemical reaction and mass transfer τ_j satisfy the following inequalities $(p_* << 1)$

$$\tau_j \omega > \tau_k \omega \to \infty,$$

where

$$\tau_k = \frac{a^2 d_0 c_v (1-p_*)}{3\rho\lambda_0 p_*}, \ \tau_j = \frac{a^2 L d_0 (1-p_*)}{3\rho\lambda_0 (g-1) T_0 p_*}.$$

Characteristic times $\tau_к$ and τ_t are related to each other by the ratio

$$\tau_t = p_*(1-p_*)^{-1}\Lambda_t \tau_k, \ \Lambda_t = \rho\frac{d_1 c_1}{d_0 c_v}.$$

The dispersion relation corresponding to the linearized system (1), (2) has the form

$$(kc_0)^2 = \omega^2 \gamma (1 + \Lambda_d - i\tau_d \omega)(1 - i\tau_d\omega)^{-1} B(\omega), \ (3)$$

$$c_0^2 = \gamma R T_0, \quad \gamma = \frac{c_p}{c_v}, \quad \Lambda_d = \rho\frac{d_1}{d_0},$$

where k is the wave number (in general, complex), ω is the real frequency, c_0 – the "frozen" speed of sound. For the kinetic regime of a chemical reaction

$$B = \left[1 + (\Lambda_t - i\tau_t\omega)(1-p_*)^{-1} + \frac{1}{i\tau_к\omega}\right]\left[\gamma + \frac{\Lambda_t - i\gamma\tau_t\omega}{1-p_*} + \frac{1}{i\tau_к\omega}\right]^{-1} \ (4)$$

We will restrict ourselves to a quadratic approximation, i.e., in the equations describing the motion of a two-phase mixture (1), we will retain only the linear and quadratic terms for the disturbances. Although the expressions for the interphase interaction force and heat fluxes (2) are obtained in a linear approximation, they can be used in the analysis of nonlinear wave processes, based on the following considerations. We will consider waves of finite amplitude, whose profile varies slightly over distances on the order of a wavelength. This variation is caused by the nonlinearity in the conservation and state equations and the nonconservative nature of the interphase interaction. If these processes are considered to have an equal effect on the wave, then including nonlinear terms in the above expressions would mean exceeding the accepted accuracy.

Denoting the quantities in a stationary gas suspension by the accepted symbols, and their perturbations in a wave by the same symbols with a prime, from (1) and (2) we obtain, with an accuracy of up to quantities of the second order of smallness in perturbations, the following system of equations

$$\frac{\partial d_0'}{dt} + d_0\left[\frac{\partial u'}{\partial x} + \rho\frac{\partial}{\partial x}(v' - u')\right] + \frac{\partial (d_0' u')}{\partial x} = 0 \quad ;$$

$$(d_0 + d_0')\frac{\partial u'}{\partial t} + d_0 u'\frac{\partial u'}{\partial x} + (\rho d_1 + \rho' d_1)\frac{\partial v'}{\partial t} + \rho d_1 v'\frac{\partial v'}{\partial x} =$$

$$= -R\frac{\partial}{\partial x}(T_0 d_0' + d_0 T_0' + d_0 T_0');$$

$$\left(d_0 + d_0^/\right)\frac{\partial T_0^/}{\partial t} + d_0 u^/ \frac{\partial T_0^/}{\partial x} - (\gamma - 1)\left[\left(T_0 + T_0^/\right)\frac{\partial d_0^/}{\partial t} + T_0 u^/ \frac{\partial d_0}{\partial x}\right] = (5)$$

$$= \frac{3\rho\lambda_0}{a^2 c_v}\left(T_1^/ - T_0^/\right) + \frac{\rho d_1}{\tau_d c_v}(u - v)^2;$$

$$\frac{\partial v^/}{\partial t} + v^/ \frac{\partial v^/}{\partial x} = \frac{u^/ - v^/}{\tau_d} \quad ;$$

$$\frac{\partial T_1^/}{\partial t} + v^/ \frac{\partial T_1^/}{\partial x} = \frac{T_0^/ - (1 - p_*)T_1^/}{\tau_t}.$$

The linearized system of equations (1) and (2) allows for unstable periodic solutions, with instability occurring when moving from high to low frequencies. Including quadratic terms in (5) leads to a transfer of energy from low-frequency oscillations upwards in the spectrum. Thus, the combined influence of nonlinearity and nonconservatism can lead to the existence of a stationary solution for system (5).

Let us consider the behavior of longitudinal acoustic vibrations in a confined heat-generating incandescent tube of length *l*. In such a system, the formation and establishment of finite-amplitude standing waves is possible due to reflection from the boundary and the superposition of perturbations traveling toward each other. However, studying the evolution of an arbitrary perturbation directly from system (5) is quite difficult. Therefore, it makes sense to reduce this system to a single evolution equation. Considering a highly dilute mixture $(\rho << 1)$, we rewrite equation (5) in dimensionless form. The first three equations in (5) will be:

$$\frac{\partial d^*}{\partial \tau} + \frac{\partial u^*}{\partial y} + \frac{\partial (d^* u^*)}{\partial y} = 0;$$

$$\frac{\partial u^*}{\partial \tau} + d^* \frac{\partial u^*}{\partial \tau} + u^* \frac{\partial u^*}{\partial y} + \Lambda_d \frac{\partial v^*}{\partial \tau} = -\frac{\partial p^*}{\partial y}; (6)$$

$$\frac{\partial T^*}{\partial \tau} + u^* \frac{\partial T^*}{\partial y} + d^* \frac{\partial T^*}{\partial \tau} - (\gamma - 1)\frac{\partial d^*}{\partial \tau} - (\gamma - 1)T^* \frac{\partial d^*}{\partial \tau} - (\gamma - 1)u^* \frac{\partial d^*}{dy} = \Lambda_t \sigma_t \left(T_1^* - T^*\right)$$

The following dimensionless variables and parameters are introduced here

$$u^* = \frac{u'}{c_*},\ p^* = \frac{(1 + \Lambda_t)(1 + \Lambda_d)}{\gamma + \Lambda_t}\frac{p'}{p},\ \tau = \frac{c_* t}{\ell},$$

$$y = \frac{x}{\ell},\ d^* = \frac{d'}{d_0},\ T^* = \frac{T'}{T_0},\ T_1^* = \frac{T_1'}{T_0},$$

$$v^* = \frac{v'}{c_*},\ \Lambda_d = \frac{\rho d_1}{d_0},\ \Lambda_t = \rho\frac{d_1 c_1}{d_0 c_v},\ \gamma = \frac{c_1}{c_v}$$

It is easy to see that the terms in equations (6) containing the variables v^* и T^* have second-order smallness in perturbations. Therefore, in the remaining equations (5), it is sufficient to retain only the linear terms:

$$\frac{\partial v^*}{\partial \tau} = \sigma_d\left(u^* - v^*\right) \quad ; \quad \sigma_d = \frac{\ell}{c_* \tau_d} \; (7)$$

$$\frac{\partial T_1^*}{\partial \tau} = \sigma_t\left(T^* - T_1^*(1 - p_*)\right); \quad \sigma_t = \frac{\ell}{c_* \tau_t}.$$

Based on the linearized system (6), we have differential relationships between pressure and velocity disturbances

$$\frac{\partial u^*}{\partial \tau} = -\frac{\partial p^*}{\partial y} \quad ; \quad \frac{\partial p^*}{\partial \tau} = -\frac{\partial u^*}{\partial y} \quad , \quad (8)$$

as well as expressions relating perturbations of pressure, density and temperature

$$d^* = p^* = \frac{1}{\gamma - 1} T^* \quad . (9)$$

Using the integral relations following from (7)

$$v^* = \sigma_d e^{-\sigma_d \tau} \int_0^\tau e^{\sigma_d \tau} u^* d\tau \quad ;$$

$$T_1^* = \sigma_t e^{-\sigma_t (1 - p_*)\tau} \int_0^\tau e^{\sigma_t (1 - p_*)\tau} T^* d\tau \quad ,$$

and also (8) and (9), within the accepted accuracy, we transform system (6) to the form

$$\frac{\partial p^*}{\partial \tau} + \frac{\partial u^*}{\partial y} = \frac{1}{2}(\gamma - 1)\frac{\partial \left(p^*\right)^2}{\partial \tau} + \frac{1}{2}\frac{\partial \left(u^*\right)^2}{\partial \tau} - \Lambda_t \sigma_t p^* + \Lambda_t \sigma_t^2 e^{-\sigma_t (1 - p_*)\tau} \int_0^\tau e^{\sigma_t (1 - p_*)\tau} p^* d\tau$$

(10)

$$\frac{\partial u^*}{\partial \tau} + \frac{\partial p^*}{\partial y} = \frac{1}{2}\frac{\partial \left(p^*\right)^2}{\partial y} - \frac{1}{2}\frac{\partial \left(u^*\right)^2}{\partial y} - \Lambda_d \sigma_d u^* + \Lambda_d \sigma_d^2 e^{-\sigma_d \tau} \int_0^\tau e^{\sigma_d \tau} u^* d\tau$$

Differentiating the first equation (10) with respect to τ, and the second with respect to y, and subtracting the second result from the first, we obtain a single nonlinear wave equation for the pressure in the furnace

$$\frac{\partial^2 p^*}{\partial \tau^2} - \frac{\partial^2 p^*}{\partial y^2} = \frac{1}{2}\left[(\gamma - 1)\frac{\partial^2 \left(p^*\right)^2}{\partial \tau^2} + \frac{\partial^2 (u^*)^2}{\partial \tau^2} + \frac{\partial^2 \left(u^*\right)^2}{\partial y^2} - \frac{\partial^2 (p^*)^2}{\partial y^2}\right] + (11)$$

$$+ \alpha_1 p^* - \alpha_2 \frac{\partial p^*}{\partial \tau} - \alpha_3 e^{-\sigma_t (1 - p_*)\tau} \int_0^\tau e^{\sigma_t (1 - p_*)\tau} p_* d\tau + \alpha_4 e^{-\sigma_d \tau} \int_0^\tau e^{\sigma_d \tau} \frac{\partial p^*}{\partial \tau} d\tau \quad ,$$

where $\alpha_1 = \Lambda_t \sigma_t^2$, $\alpha_2 = \Lambda_t \sigma_t + \Lambda_d \sigma_d$,

$$\alpha_3 = \Lambda_t \sigma_t^3 (1 - p_*) \quad , \quad \alpha_4 = \Lambda_d \sigma_d^2 .$$

The left-hand side of (11) is the standard wave-level operator, the right-hand side consists of nonlinear square terms and terms describing energy pumping, its dissipation, and distributed dispersion effects. The latter also have second-order smallness in perturbations due to the fact that the coefficients $\alpha_i << 1$, i = 1, 2, 3, 4.

For a linear conservative system, instead of (11) we have the usual wave equation

$$\frac{\partial^2 p^*}{\partial \tau^2} - \frac{\partial^2 p^*}{\partial y^2} = 0 \quad (12)$$

The solution of this equation in a limited volume can be represented as a sum of standing waves

$$p^* = \sum_{n=-\infty}^{\infty} \Phi_n e^{-i\omega_n \tau} \cos(\kappa_{|n|} y + \varphi_{|n|}) \quad ,$$

$$\Phi_n^* = \Phi_{-n} \quad , \quad -\omega_n = \omega_{-n} \quad , \quad \omega_n^2 = k_n^2 \quad ,$$

where Φ_n - arbitrary amplitude, ω_n - natural frequency of mode with number n , φ_n - phase.

It was shown above that the terms on the right-hand side of equation (11) are significantly smaller than those on the left-hand side. Therefore, we will seek a solution to equation (11) as a series in terms of the eigenmodes of the generating linear conservative equation, assuming that, as a result of nonlinear interaction, dissipation, and energy pumping, the amplitudes of the standing waves are slowly varying functions of time:

$$p^* = \sum_{n=-\infty}^{\infty} \Phi_n(\mu\tau) e^{-i\omega_n \tau} \cos(\kappa_{|n|} y + \varphi_{|n|}), \quad \mu << 1, \quad (13)$$

$$u^* = i \sum_{n=-\infty}^{\infty} \mathrm{sign}(n) \Phi_n(\mu\tau) e^{-i\omega_n \tau} \sin(\kappa_{|n|} y + \varphi_{|n|}) .$$

Substituting expressions (13) into equation (11), we obtain

$$\sum_{n=1}^{\infty} \omega_n \left(\frac{dF_n}{d\tau} - (\gamma_n - i(\delta_n + \varepsilon_n)) F_n \right) e^{-i\omega_n \tau} \cos\chi_n = \quad (14)$$

$$= -\frac{1}{8} i\gamma \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \Big[(\omega_m + \omega_n)^2 F_n F_m e^{-i(\omega_n + \omega_m)\tau} \cos(\chi_m + \chi_n) \Big] +$$

$$+ (\omega_m - \omega_n)^2 F_m F_n^* e^{-i(\omega_m - \omega_n)\tau} \cos(\chi_m - \chi_n) \quad .$$

Here

$$\omega_n = \kappa_n = \pi n + \varepsilon_n \quad , \quad \chi_n = \pi n y + \psi_n \quad ,$$

$$\Psi_n = \varepsilon_n y + \varphi_n \quad , \quad F_n = \Phi_n e^{-i\varepsilon_n \tau} \quad ,$$

$$\gamma_n = \frac{1}{2} \left(\frac{\Lambda_t \sigma_t^3 (1 - p_*)}{\sigma_t^2 (1 - p_*)^2 + \omega_n^2} + \frac{\Lambda_d \sigma_d^3}{\sigma_d^2 + \omega_n^2} - \Lambda_d \sigma_d - \Lambda_t \sigma_t \right),$$

$$\delta_n = \frac{1}{2} \left(\frac{\Lambda_t \sigma_t^4 (1 - p_*)^2}{\omega_n \left(\sigma_t^2 (1 - p_*)^2 + \omega_n^2 \right)} - \frac{\Lambda_t \sigma_t^2}{\omega_n} - \frac{\Lambda_d \sigma_d^2 \omega_n}{\sigma_d^2 + \omega_n^2} \right).$$

We assume that the ends of the volume are close to acoustically closed, i.e. $|\varphi_n| << 1$ and $|\varepsilon_n| << 1$. Then equation (14) can be reduced to an infinite system of differential equations for complex amplitudes

$$\frac{dF_n}{d\tau} - \left(\gamma_n - i(\delta_n + \varepsilon_n)\right)F_n = -\frac{i\gamma\pi n}{8}\left\{2\sum_{m=1}^{\infty} F_{m+n}F_m^* + \sum_{m=1}^{n} F_m F_{n-m}\right\} \quad ; \quad n = 1,2,3...$$

(15)

Let us introduce real amplitudes and phases into consideration, i.e., let us set $F_n = p_n e^{i\theta n}$ $(n = 1,2,...)$, where p_n - amplitude, θ_n - phase. Separating the real and imaginary parts in (15), we obtain a system of equations describing the nonlinear interaction of standing waves in the quadratic approximation.

$$\frac{dp_n}{d\tau} = \gamma_n p_n + \frac{\gamma\pi n}{8}\left\{2\sum_{m=1}^{\infty} p_{n+m}p_m \sin(\theta_{m+n} - \theta_m - \theta_n) - \sum_{m=1}^{n} p_m p_{n-m} \sin(\theta_n - \theta_m - \theta_{n-m})\right\} \quad (16)$$

$$p_n \frac{d\theta_n}{d\tau} = -(\delta_n + \varepsilon_n)p_n - \frac{\gamma\pi n}{8}\left\{2\sum_{m=1}^{\infty} p_{n+m}p_m \cos(\theta_{m+n} - \theta_m - \theta_n) - \sum_{m=1}^{n} p_m p_{n-m} \cos(\theta_n - \theta_m - \theta_{n-m})\right\}$$

$$n = 1,2,...$$

Next, we restrict ourselves to the case of interaction of four vibration modes (n = 1, 2, 3, 4). A necessary condition for the existence of steady-state amplitudes of sound waves different from zero is the presence of at least one mode that decays according to the given approximation, i.e., in the set γ_n (n = 1, 2, ,3, 4) must contain quantities of different signs. Let us consider the case of a shallow entry into the instability region ($\gamma_1 > 0$, $\gamma_2 < 0$, $|\gamma_1| << 1$), when the amplitude of only the fundamental tone increases, while the remaining overtones are damped (since in real processes, high harmonics are characterized, all other things being equal, by increased energy dissipation). The stationary solution of equation (16) is found from a system of nonlinear algebraic equations for p_n and Δ_n

$$\gamma_n p_n + \frac{\gamma\pi n}{8}\left\{2\sum_{m=1}^{4-n} p_{n+m}p_m \sin\Delta_{mn} - \sum_{m=1}^{n} p_m p_{n-m} \sin\Delta_{(n-m)m}\right\} = 0$$

$$g_{\kappa+1} - g_k - g_1 = 0, \quad g_4 - 2g_2 = 0, \quad \kappa = 1,2,3 .$$

Here

$$g_n = -(\delta_n + \varepsilon_n) - \frac{\gamma\pi n}{8}\left\{2\sum_{m=1}^{4-n} \frac{p_n + mp_m}{p_n} \sin\Delta_{mn} - \sum_{m=1}^{n} \frac{p_m p_{n-m}}{p_n} \sin\Delta_{(n-m)m}\right\} = 0 \quad ,$$

$$\Delta_\kappa = \theta_{k+1} - \theta_k - \theta_1 (k = 1,2,3) \quad , \quad \Delta_4 = \theta_4 - 2\theta_2 .$$

Using Seidel's method, we find the steady-state oscillation amplitudes with and without dispersion. Thus, dispersion, caused by the mismatch between the average temperatures and phase velocities of the gas suspension, leads to an increase in the amplitudes of the first overtones and a decrease in the amplitudes of subsequent harmonics. The latter is explained by the fact that the presence of dispersion in the system disrupts internal resonances and, consequently, limits the transfer of energy up the spectrum.

Conclusion. In this article, we have derived a single nonlinear wave equation describing the evolution of pressure in a furnace to study the behavior of acoustic disturbances in a confined volume of a burning gas suspension. The assumption that nonlinearity, dispersion, and nonconservative nature of the oscillations have a small effect on wave amplitudes allowed us to use the eigenmode expansion method for a linear conservative problem to solve the resulting equation. This procedure reduced the original wave equation to an infinite chain of ordinary differential equations for complex amplitudes. The steady-state amplitudes of the standing waves were determined.

Список источников

1. Раушенбах Б.В. Вибрационное горение. - М.: Физматгиз, 1961. - 500с.
2. Рогинский О.Г. О вибрационном горении. Обзор // Акустический журнал. – 1967. –Т.7, вып. 2. – С. 131-155.
3. Хзмалян Д.М., Каган Я.А. Теория горения и топочные устройства. – М.: Энергия, 1976 – 488 с.
4. Зельдович Я.Б., Баренблотт Г.И., Либрович В.Б., Махвиладзе Г.М. Математическая теория горения и взрыва. – М.: Наука, 1984. – 374 с.
5. Основы практической теории горения. Под ред. Померанцева В.В. - Л.: Энергоатомиздат , 1986. - 312с.
6. Аракелян Э.К., Янукян Э.Г. Горение полидисперсного топлива // Известия РАН. Энергетика. – Москва, 2002, №2.- С. 60-67.
7. Янукян Э.Г. Волновые режимы горения в конденсированных средах // Журнал «Обозрение прикладной и промышленной математики». – Москва, 2004, т.11, С. 130.
8. Yanukyan E. G. Computational modeling of controlled unsteady combustion for dispersed fuels // Book of abstracts of the Fifth World Congress on Computational Mechanics. - Vienna, Austria, 2002, Vol. 2. - P. 584.
9. Борисов А.А. Распространение возмущений в двухфазных химически реагирующих средах // Нелинейные волновые процессы в двухфазных средах.- ИТФ СОАН СССР. Новосибирск.- 1977.-с.128-142.
10. Борисов А.А., Вахгельт А.Ф. Эволюция волн в двухфазных химически реагирующих средах. – Деп. ВИНИТИ. №5576.-1984.-20с.
11. Хитрин Л.Н. Физика горения и взрыва. – М.: Изд-во МГУ, 1957. – 443с.

References

1. Raushenbakh B.V. Vibratsionnoe gorenie. - M.: Fizmatgiz, 1961. - 500s.
2. Roginskii O.G. O vibratsionnom gorenii. Obzor // Akusticheskii zhurnal. – 1967. –T.7, vyp. 2. – S. 131-155.
3. Khzmalyan D.M., Kagan YA.A. Teoriya goreniya i topochnye ustroistva. – M.: Ehnergiya, 1976 – 488 s.
4. Zel'dovich YA.B., Barenblott G.I., Librovich V.B., Makhviladze G.M. Matematicheskaya teoriya goreniya i vzryva. – M.: Nauka, 1984. – 374 s.
5. Osnovy prakticheskoi teorii goreniya. Pod red. Pomerantseva V.V. - L.: Ehnergoatomizdat , 1986. - 312s.
6. Arakelyan EH.K., Yanukyan EH.G. Gorenie polidispersnogo topliva // Izvestiya RAN. Ehnergetika. – Moskva, 2002, №2.- S. 60-67.
7. Yanukyan EH.G. Volnovye rezhimy goreniya v kondensirovannykh sredakh // Zhurnal «Obozrenie prikladnoi i promyshlennoi matematikI». – Moskva, 2004, t.11, S. 130.
8. Yanukyan E. G. Computational modeling of controlled unsteady combustion for dispersed fuels // Book of abstracts of the Fifth World Congress on Computational Mechanics. - Vienna, Austria, 2002, Vol. 2. - P. 584.
9. Borisov A.A. Rasprostranenie vozmushchenii v dvukhfaznykh khimicheski reagiruyushchikh sredakh // Nelineinye volnovye protsessy v dvukhfaznykh sredakh.- ITF SOAN SSSR. Novosibirsk.- 1977.-s.128-142.
10. Borisov A.A., Vakhgel't A.F. Ehvolyutsiya voln v dvukhfaznykh khimicheski reagiruyushchikh sredakh. – Dep. VINITI. №5576.-1984.-20s.
11. Khitrin L.N. Fizika goreniya i vzryva. – M.: Izd-vo MGU, 1957. – 443s.

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