Современная наука и инновации. 2025. № 3. С 39-51.
ТЕХНИЧЕСКИЕ НАУКИ
ИНФОРМАТИКА, ВЫЧИСЛИТЕЛЬНАЯ ТЕХНИКА И УПРАВЛЕНИЕ

Modern Science and Innovations. 2025;(3):39-51.
TECHNICAL SCIENCE
INFORMATICS, COMPUTER ENGINEERING AND MANAGEMENT

Научная статья
УДК 621.391
https://doi.org/10.37493/2307-910X.2025.3.4

Равновесные состояния плазмы в потенциальном внешнем поле с эквипотенциальными магнитными поверхностями

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*** Автор, ответственный за переписку**

Аннотация. Введение. *В данной работе рассмотрены равновесные состояния плазмы в потенциальном внешнем магнитном поле с эквипотенциальными магнитными поверхностями. Приведена теория, описывается уравнение для однопараметрического семейства магнитных поверхностей. Приведено общее решение для системы уравнений магнитной гидродинамики и уравнения, описывающее семейство однопараметрических уравнений. Поведение плазмы во многом определяется пространственной структурой, ограничивающей магнитное поле, то есть* криволинейными координатами E_3. *В результате доказательства леммы получены выражения для давления, плотности, потенциала магнитного поля. При произвольных функциях (b) в E_3. Доказана теорема, что в системе координат (x'), не содержит производную по переменной x', эта переменная входит как параметр в уравнения.* ***Материалы и методы.*** *Аналитически получены общие решения для равновесных состояний плазмы во внешнем потенциальном поле в криволинейных координатах.* ***Результаты и обсуждение.*** *В данной статье рассмотрены возможности из полученной теории для построения решений уравнений, позволяющие выбрать в качестве независимых переменных произвольные функции. С помощью математического пакета Maple приведены примеры графического представления решений для давления, плотности, силовые линии магнитного поля, квадрата напряженности магнитного поля для равновесных состояний плазмы в потенциальном внешнем магнитном поле с эквипотенциальными магнитными поверхностями.* ***Заключение.*** *Результаты исследования можно использовать при изучении равновесных состояний плазмы в потенциальном внешнем магнитном поле.*

Ключевые слова: однопараметрическое семейство магнитных поверхностей, компоненты тензора Леви-Чивита, эйлеровый потенциал магнитного поля, плотность, давление, напряженность магнитного поля, редуцирование условий равновесия, равновесные состояния плазмы, эквипотенциальные магнитные поверхности.

Для цитирования: *Джимбеева Л. Н., Шаповалов А. В. Равновесные состояния плазмы в потенциальном внешнем поле с эквипотенциальными магнитными поверхностями // Современная наука и инновации. 2025. №3. С. 39-51. https://doi.org/10.37493/2307-910X.2025.3.4*

Research article

The equilibrium states of plasma in a potential external field with equipotential magnetic surfaces

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Abstract. Introduction. *In this paper, the equilibrium states of plasma in a potential external magnetic field with equipotential magnetic surfaces are considered. A theory is given and an equation for a one-parameter family of magnetic surfaces is described. A general solution for the system of equations of magnetic hydrodynamics and an equation describing a family of one-parameter equations are given. The behavior of the plasma is largely determined by the spatial structure that limits the magnetic field, that is, the curved coordinates E_3. As a result of the proof of the lemma, expressions for pressure, density, and magnetic field potential are obtained. For arbitrary functions (b) in E_3. The theorem is proved that in the coordinate system (x'), it does not contain a derivative of the variable x', this variable is included as a parameter in the equations.* ***Materials and methods.*** *General solutions for the equilibrium states of plasma in an external potential field in curvilinear coordinates have been obtained analytically.* ***Results and discussion.*** *In this paper we consider the possibilities from the obtained theory for constructing solutions to the equations which allow us to choose arbitrary functions in qualitatively independent variables. Using the mathematical package Maple, examples of graphical representation of solutions for pressure, density, magnetic field force lines and magnetic field strength square for equilibrium states of plasma in an external potential magnetic field with equipotential magnetic surfaces are given.* ***Conclusion.*** *The results of the study can be used to study the equilibrium states of plasma in a potential external magnetic field.*

Key words: one-parameter family of magnetic surfaces, components of the Levi-Civita tensor, Eulerian magnetic field potential, density, pressure, magnetic field strength, reduction of equilibrium conditions, plasma equilibrium states, equipotential magnetic surfaces.

For citation: *Dzhimbeeva LN, Shapovalov AV. Equilibrium states of plasma in a potential external field with equipotential magnetic surfaces. Modern Science and Innovations. 2025;(3):39-51. (In Russ.). https://doi.org/10.37493/2307-910X.2025.3.4*

Introduction. In the introduction we will consider the theory of the issue, most of which is presented in the works [3, 5, 6, 7, 8, 12] and is not presented in this paper. In the works [1, 2, 4, 9, 10, 11] the issues of plasma physics related to the problem under consideration in this article are considered.

Let b (x) = c be a one-parameter family of magnetic surfaces, thus we have

$$\left(\boldsymbol{H}(x), \nabla \boldsymbol{b}(x)\right) = 0,\ \nabla b(x) \neq 0 \quad (1)$$

Together with (1) it is necessary to integrate the system

$$div\boldsymbol{H} = 0 \quad (2\text{ a })$$

$$[rot\boldsymbol{H}, \boldsymbol{H}] = \nabla p + \rho \nabla u; \quad (2\text{ b })$$

Here p is the plasma pressure, u is the potential of the external field, $\sqrt{4\pi}\,\boldsymbol{H}$and is the magnetic field strength. Now we have

$$\boldsymbol{H}(x) = [\nabla a(x), \nabla b(x)]; \quad [\nabla a, \nabla b] \neq 0; \quad (3)$$

This is the general solution of equations (1) and (2 a , 2 b).

Let us consider equation (2 b) using the following equality

$$[rot\boldsymbol{H}, \boldsymbol{H}] = (\nabla b, rot[\nabla a, \nabla b])\nabla a - (\nabla a, rot[\nabla a, \nabla b])\nabla b.$$

Equation (2 b) can be written as

$$dp = (\nabla b, rot[\nabla a, \nabla b])da - (\nabla a, rot[\nabla a, \nabla b])db - \rho du$$
(.4)

Let's consider the available options.

A. Now , $(\nabla u, [\nabla a, \nabla b]) \neq 0$;as independent variables, E_3we can select the quantities (a , b , u) .

From (4) it follows

$$p = p(a, u, v), \quad \rho = -p_{,u}; (5\ a)$$
$$p_{,a} = (\nabla v, rot[\nabla a, \nabla v]); \quad p_{,v} = (\nabla a, rot[\nabla a, \nabla v]). (5b)$$

The following lemma is used further.

Lemma. Let x'there be curvilinear coordinates E_3now $\frac{\partial}{\partial x'^i} = \frac{1}{2}\varepsilon'_{ijk}(x')([\nabla x'^j, \nabla x'^k], \nabla)$,*here* ε_{ijk}*are* the components of the tensor Levi-Civita in the coordinate system (x').

Proof. Let be $x$the Cartesian coordinates in E_3,multiplying the equality

$$\frac{\partial}{\partial x^q} = \frac{\partial x'^s}{\partial x^q}\frac{\partial}{\partial x'^s}$$

to the expression

$$\frac{\partial x'^r}{\partial x^n}\frac{\partial x'^p}{\partial x^m}\varepsilon^{nmq}$$

and taking into account the tensor nature of the symbol ε :

$$\varepsilon^{nmp}\frac{\partial x'^q}{\partial x^m}\frac{\partial x'^s}{\partial x^q}\frac{\partial x'^r}{\partial x^n} = \varepsilon'^{prs}(x'),$$

we get

$$([\nabla x'^p, \nabla x'^r], \nabla) = \varepsilon'^{prs}(x')\frac{\partial}{\partial x'^s}$$

Multiplying this equality by $\varepsilon'_{ipr}(x')$and taking into account that

$$\varepsilon'_{ipr}(x')\varepsilon'^{prs}(x') = 2\delta_i^p,$$

we get what we need.

Note: Let's write down, as an example,$\partial/\partial x'^1$:

$$\frac{\partial}{\partial x'^1} = \sqrt{g'(x')}([\nabla x'^2, \nabla x'^3], \nabla),$$
$$\sqrt{g'(x')} = (\nabla x'^1, [\nabla x'^2, \nabla x'^3])^{-1}$$

Using this lemma and remark, it is easy to write down the identities

$$\frac{\partial}{\partial a} = -(\nabla v, [\nabla a, \nabla u])^{-1}(\nabla v, [\nabla u, \nabla]),$$
$$\frac{\partial}{\partial v} = (\nabla v, [\nabla a, \nabla u])^{-1}(\nabla a, [\nabla u, \nabla]).$$

Now the compatibility condition of the system (5 b) can be written as

$$(\nabla a, [\nabla u, \nabla])(\nabla b, rot[\nabla a, \nabla b]) = (\nabla b, [\nabla u, \nabla])(\nabla a, rot[\nabla a, \nabla b]); (6)$$

From this equation we find (*a)* , specifying arbitrarily the function *b* . Now for the pressure *p* we have

$$p = \int (\nabla v, rot[\nabla a, \nabla b])da - (\nabla a, rot[\nabla a, \nabla b])db (7)$$

Thus, formulas (3), (5 a), (6), (7) provide a solution to the problem, while (*b)* is an arbitrarily given function в E_3.

B . $(\nabla a, [\nabla b, \nabla u]) = 0$;without losing the generality of the arguments, we set b = u **.** Now from equation (4) we have

$$p = p(a, u),$$

$$\rho = -p_{,u} - (\nabla a, rot[\nabla a, \nabla u]),$$

(8a)

$$p_{,a}(a,u) = (\nabla u, rot[\nabla a, \nabla u])\textit{(8b)}$$

From equations (8) with an arbitrarily given function *p (a , u)* we find the potential *(a), we will do* A few comments. Let's move from Cartesian coordinates (x)to coordinates (x'):

$$x'^1 = x^1, \quad x'^2 = x^2, \quad x'^3 = x^3 \qquad (9)$$

Function $g' = \det(g'_u(u'))$. We also note that the scalar potential of the external field in coordinates $(x)$is given by the function $u(x)$,and in coordinates $(x')$by virtue of (9) by the function $u'(x') = x'^3$,thus, $u'_{,n} = \delta_{3n}$. Next, the structure of equation (8 b) in coordinates is obtained (x').

Main part. Here, as before, we use the following notation: a $E_3 \to R$**is** the Euler potential of the magnetic field; **u** is $E_3 \to R$the potential of the external field; **p** is $E_3 \to R$the pressure in the plasma.

Let us consider the following equations (8).

$$\boldsymbol{H} = [\boldsymbol{\nabla}(\boldsymbol{a}), \boldsymbol{\nabla}(\boldsymbol{u})];$$

(10)

$$\boldsymbol{\rho} = -\boldsymbol{p}_{,u}(\boldsymbol{a},\boldsymbol{u}) + (\boldsymbol{\nabla}(\boldsymbol{a}), \boldsymbol{rot}[\boldsymbol{\nabla}(\boldsymbol{u}), \boldsymbol{\nabla}(\boldsymbol{a})]); \qquad (11)$$

$$p_{,a}(\boldsymbol{a},\boldsymbol{u}) = -(\boldsymbol{\nabla}(\boldsymbol{u}), \boldsymbol{rot}[\boldsymbol{\nabla}(\boldsymbol{u}), \boldsymbol{\nabla}(\boldsymbol{a})]). \mathbf{(12)}$$

Equation (12) can be written as

$$p_{,a}(a,u) = div([\nabla(u), [\nabla(u), \nabla(a)]). (13)$$

Next we deal with a potential of the form $u(x,y,z) = z$.Now equation (13) takes the form

$$p_{,a}(a,u) = -a_{,xx}(x,y,z) - a_{,yy}(14)$$

Below we present a particular solution to the equation with a specific type of pressure.

p. 1. Let

$$p(a,z) = -exp\,(a) + P(z);$$

Here P(z) is an arbitrary function; now we can find the general solution of equation (14). Note that the variable z is a parameter. We introduce new variables s, t: $x = s * \cos(q)$, $y = r * \sin(q)$. In the new variables, equation (14) has the form$a_{,st}$ $(s,t,z) = \exp(a)$.

The general solution to this equation is:

$$a = \ln\,(2 * A(s) * B(t)) *(\mathrm{A}\,(\,\mathrm{s}\,)+B(t)^{(-2)});$$

here A, B are arbitrary functions of their variables and the parameter z. Since the potential $a$is a real function, we should set the function B(t) = conjugate A(s). Now the potential has the form

$$a = \ln(2 * |A'(x)|^2) * (2 * Re\, A)^{-2} \qquad (15)$$

Next, we will consider several implementations of the function$A(s)$.

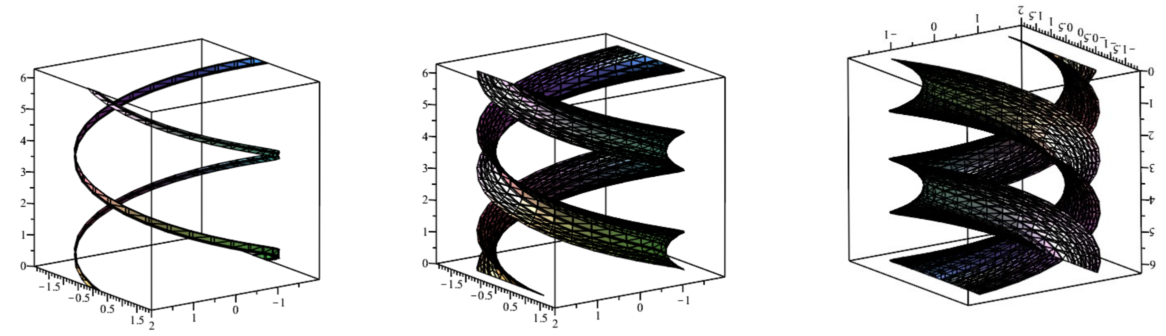
n .1.1. Let the function have the form $\boldsymbol{A(s,z) = exp(i * h(z) * s^n) \;\; + v(z) * 2^{(2-n)}}$; $\boldsymbol{h, v}$ - arbitrary real functions. The potential has the form $a = ln(2 * n^2 * r^{2*n-2} * (r^n * \cos\,(n * q + h) + v)^{-2})$; here and below r, q– polar coordinates:

$$x = r * \cos(q)\,, \quad y = r * \sin(q).$$

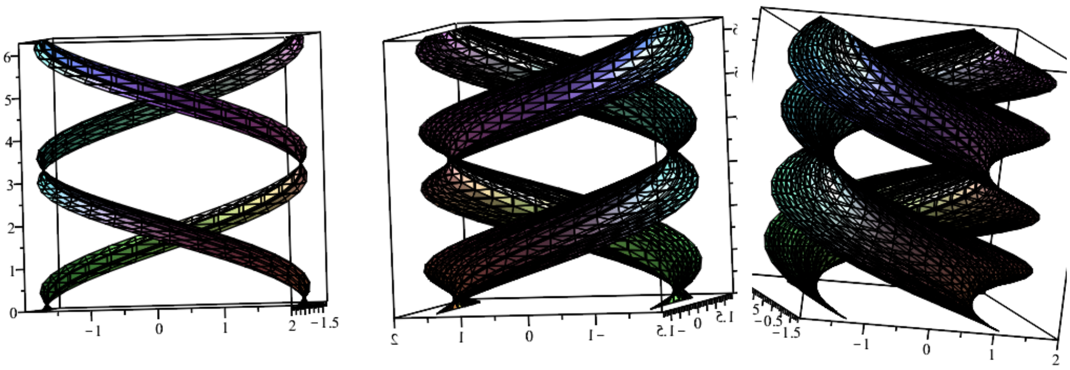
Let us give some specific examples:

Example 1 for $a \coloneqq (r,q,z) \to \ln(2) - \ln(r^2 * \cos(q+z)^2)\,; a1 \coloneqq a_{,r}\,;\; a2 \coloneqq a_{,q}),$

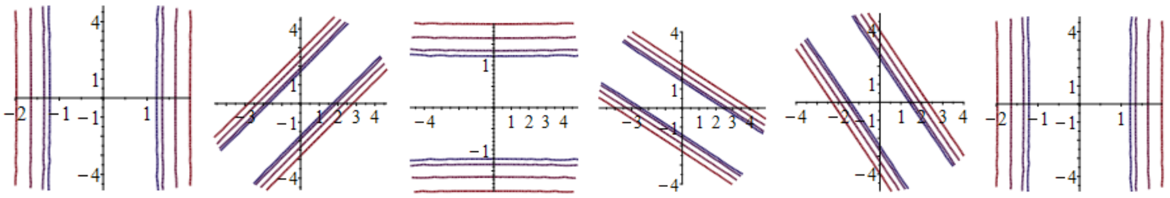
Let us plot graphs for the following values of a: = c: c= -0.67, -0.3, 0.1.



Let us construct graphs for the square of the stress $h \equiv H^2 = c$, with the following values **c** = 1.1, 1.6, 3.



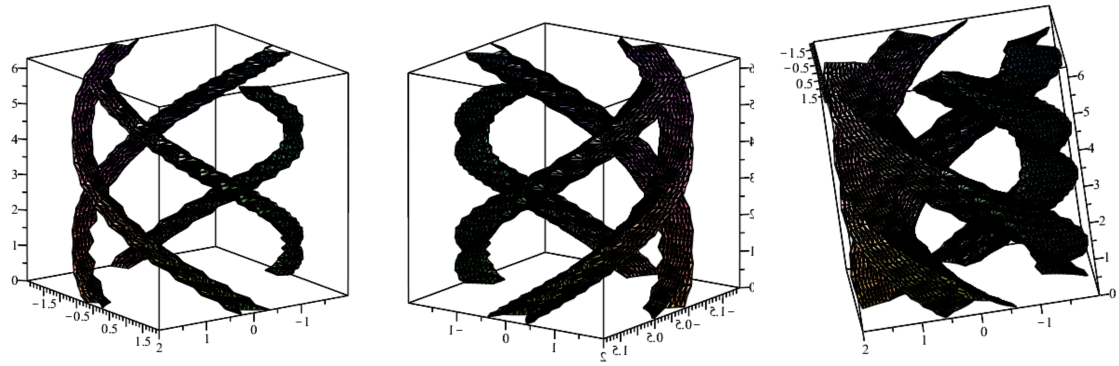
Let us construct graphs of magnetic field lines { a := c (c =-0.67, -0.3, 0.1); z = u (u =0, Pi /4, Pi /2)}:



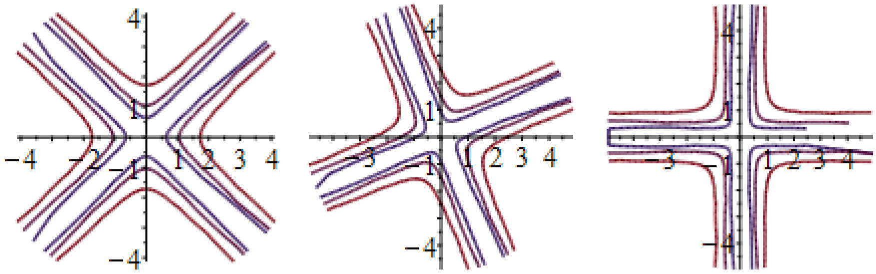
Example 2 .

$a \coloneqq (r,q,z) \rightarrow \ln(8) - \ln(r^2 * cos(2*q+z)^2)$; $a1 \coloneqq diff(a(r,q,z),r)$; $a2 \coloneqq diff\ (a(r,q,z),q)$.);

Graphs for the square of the stress $H^2 = c$, c = 1.5, 2, 3.5.

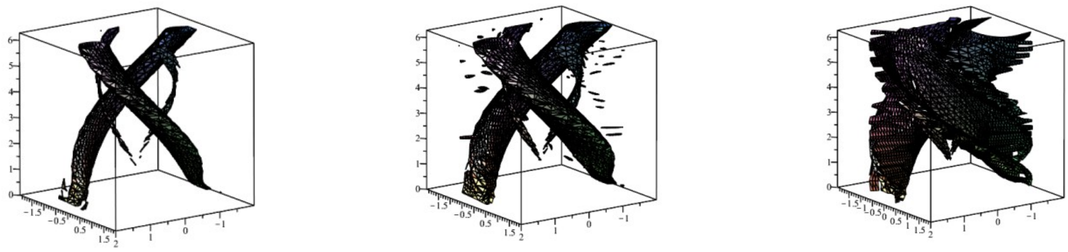


Graphs of magnetic field lines { a := c : (c =-0.67,-0.3,0.1); z = u (u =0, Pi /4, Pi /2)}.

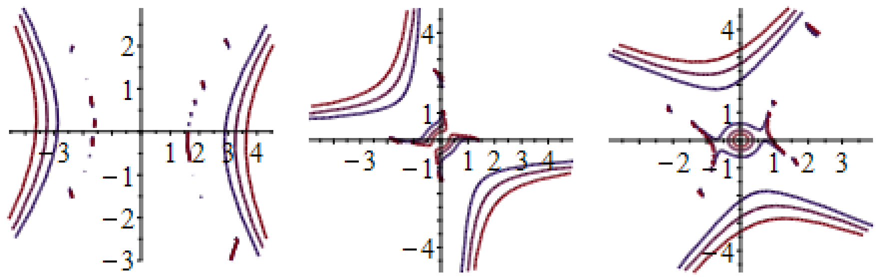


Example $\boldsymbol{a \coloneqq (r,q,z) \to ln\,(8\cdot r^2) - ln(r^2 * cos(2*q+z) + ln(z))^{3.2}}$; $\boldsymbol{a1 \coloneqq diff(a(r,q,z).x)}$; $\boldsymbol{a2 \coloneqq diff(a(r,q,z).q)}$.

Graphs for the square of the magnetic field strength $H^2 = c$, c = 1, 2, 5.



Graphs of magnetic field lines { a = c (c =-1, 0, 1); z = u (u =0, Pi /2, 5* Pi /6)}.



Let's consider a function in the form $A \coloneqq \exp(2*s) + v(z)$.

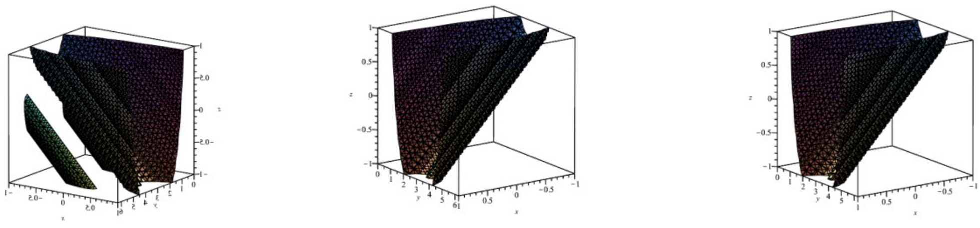
The potential is of the form $a := \ln(2) - \ln\,((\cos(y) + v(z) * \exp(-x))^2$.

Specific examples of this situation:

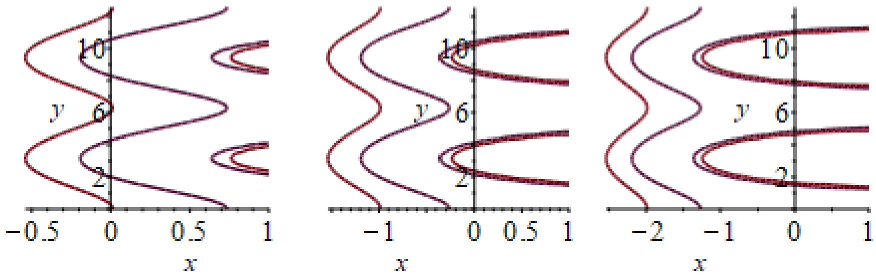
Example 4.2 $\boldsymbol{a \coloneqq (x,y,z) \to ln(2) - ln\,(cos(y) + exp(-x-z))}$

$\boldsymbol{a1 \coloneqq diff(a(x,y,z),x); a2 \coloneqq diff(a(x,y,z),y)}$.

Let us construct graphs of the field strength $H^2 = c$, c = 0.5, 2, 5. .



Graphs of magnetic field lines { a : = c (c =-1, 0, 1); z = u (u =-1, 0, 1)}.

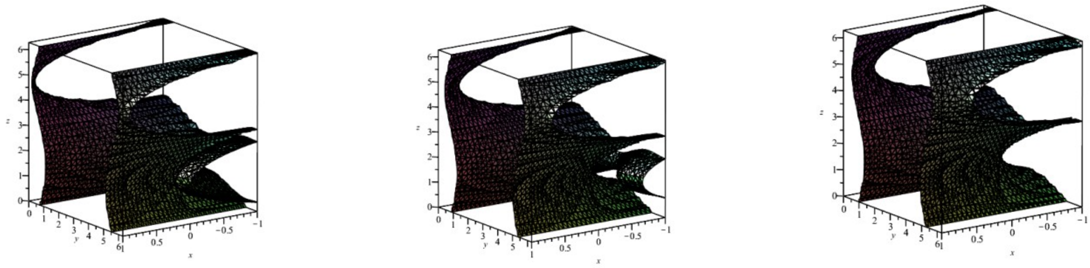


Example 5.

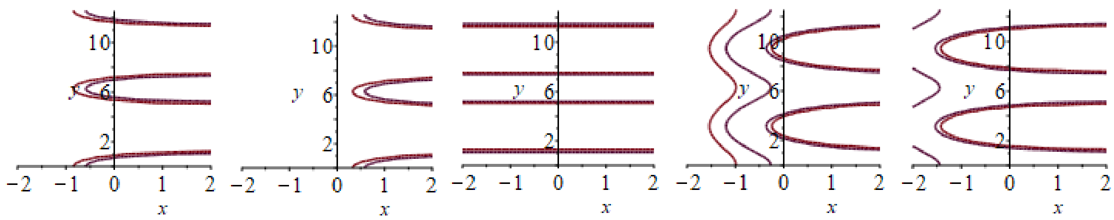
$\boldsymbol{a \coloneqq (x.y.z) \ \rightarrow ln(2) - ln(cos(y) + sin(z) * exp(-x))^2}$

$a1 \coloneqq diff(a(x,y,z),x); a2 \coloneqq diff(x,y,z),y)$

Graphs for H ^{2} = c , c = 0.5, 2, 5.



Graphs of magnetic field lines { a = c (c =-1, 0, 1); z = u (u =-9* Pi /10, - Pi /2.0, Pi /2, 9* Pi /10)}.



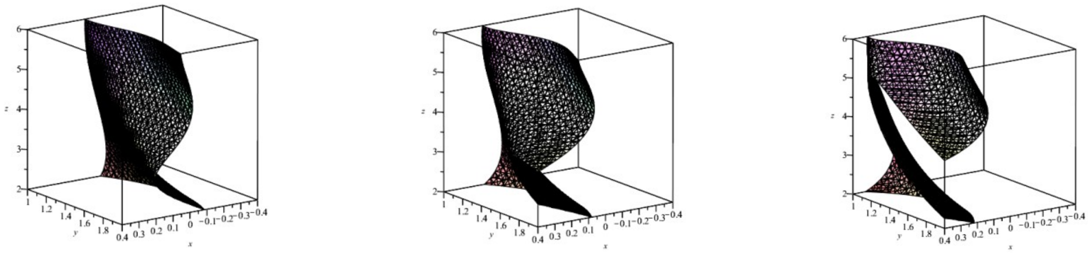
Let's consider a function in the form $\boldsymbol{A = exp(2 * g(z) * cos\big(h(z) + r * sin(h)\big)) + v(z)}$.

Let's give examples of this situation:

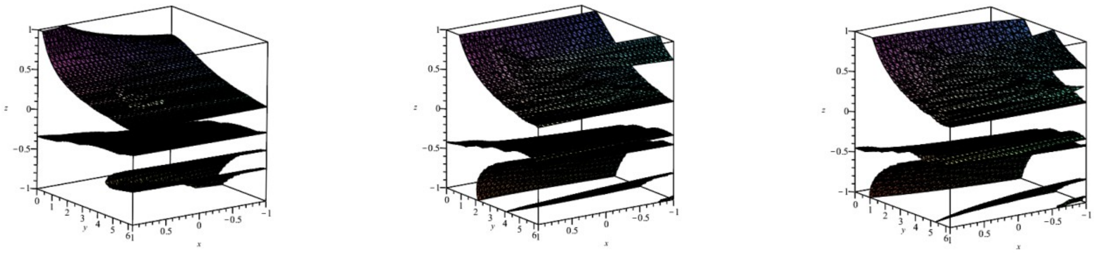
Example 6.

$\boldsymbol{a \coloneqq (x.y.z) \rightarrow ln\left(2 * z^2 * \frac{exp(2*z*x)}{exp(z*x)} * cos(z*y) + z\right)^2}$ $a1 \coloneqq diff(a(x,y,z),a); a2 \coloneqq diff(a(x,y,z),x);\ \ a2 \coloneqq diff(a(x,y,z).y)$.

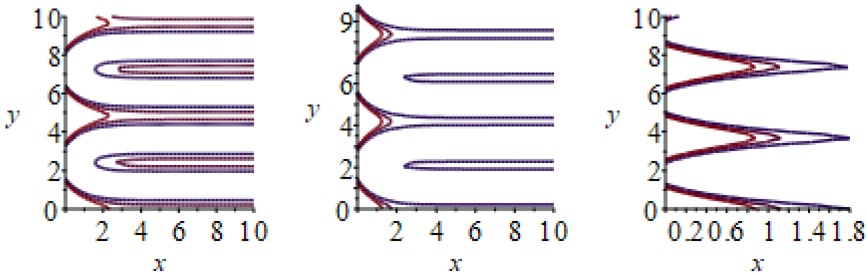
Graphs for potential a:=c: c=1, 2, 3.



Graphs for the square of the field strength H ^{2} = c , c = 0.5, 3, 7.

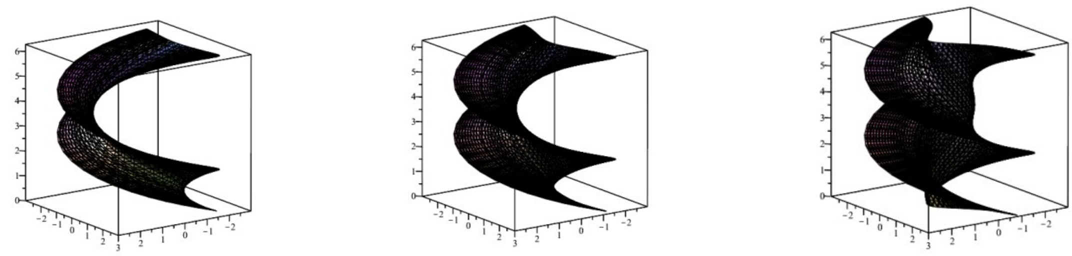


Graphs of magnetic field lines { a := c (c =1.1, 1.3, 1.6); z = u (u =1.3, 1.5, 1.7)}.

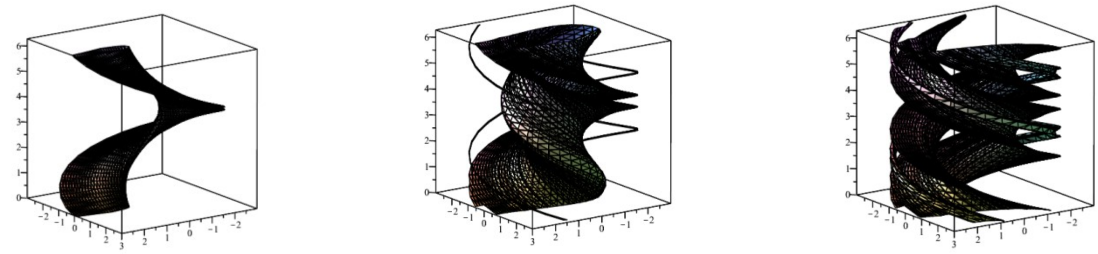


Example 7. $a := (r, q, z) \rightarrow ln(2 * exp(2 * r * cos(q + z))/(exp(r * cos(q + z)) * cosr*sinq+z+1^{2}$); *a1:=diff(ar,q,z;a2:=dyffar,q,z.q.*

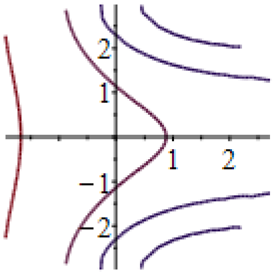
Let's plot the graphs a:=c: c=-3, -1.5, 0. .

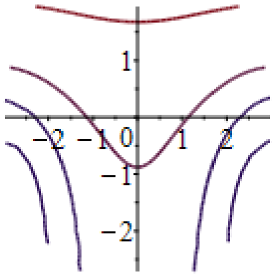


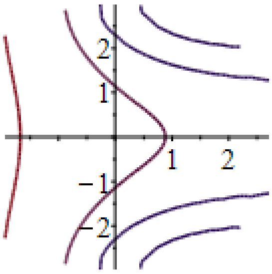
Let's plot graphs for the square of the magnetic field strength: H ^{2} = c , c = 0.5, 3, 7.



Let us construct graphs of magnetic field lines { a : = c (c =-3, 0 ,3); z = u (u =0, Pi /2, 2* Pi)}.







p.2. Let$\boldsymbol{p(a,z) = a + P(z)}$,

where is $P(z)$ − an arbitrary function; one can find the general solution of equation (15):

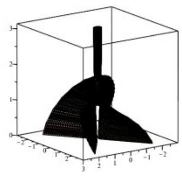
$a := \frac{r^2}{4} + 2 * Re\,(A(s))$;Here is $A(s)$ − пan arbitrary function.

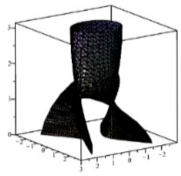
Let's give specific examples of this situation.

Example 8. Let's consider$a := (r,q,z) \to \frac{r^2}{4} + r^2 * \exp(-z) * \cos(2 * q + z)$;

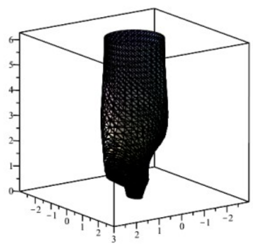
$a1 := diff(a(r,q,z),r)$; $a2 := diff(a(r,q,z),q)$.

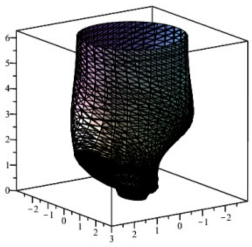
Let's plot graphs for a=c: c=0.01, 0.1, 0.4, 1 .

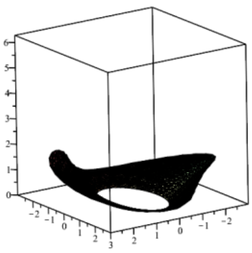




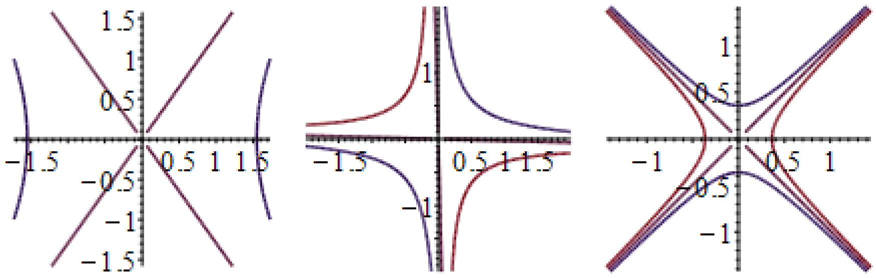
Let us construct graphs of the square of the magnetic field strength $H^2 = c$, c = 0.3, 1, 7.







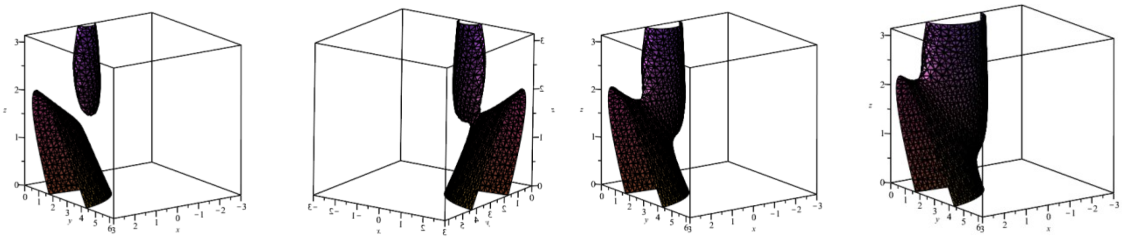
Let us plot the graphs of magnetic field lines { a := c (c =-3,0,3); z = u (u =0,- Pi /2,- Pi)}



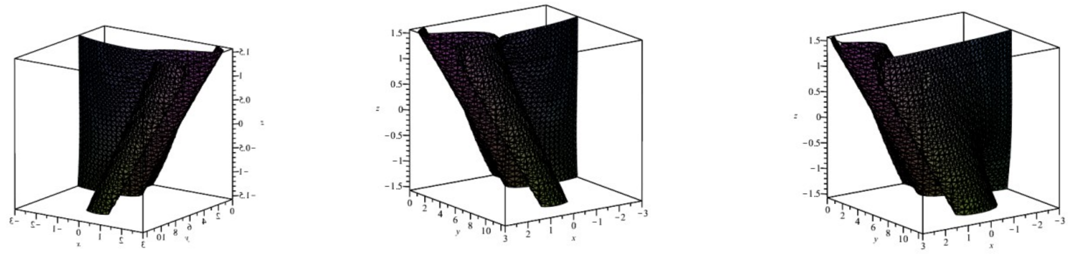
Example 9. $\boldsymbol{a := (x, y, z) \to \frac{x^2 + y^2}{4} + exp(x - z) * cos(y + z);}$

$$\boldsymbol{a1 := diff(a(x, y, z), x);\ \ a2 := diff(a(x, y, z), y).}$$

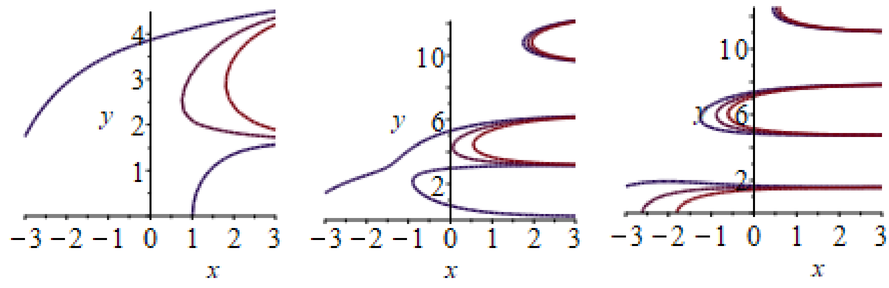
Let us plot graphs for the corresponding values of a=c: c=-0.01, 0.015, 0.1, 0.3.



Let us construct graphs of the square of the magnetic field strength $H^2 = c$, c = 5.5, 6.5, 12. .



Let us construct graphs of magnetic field lines { a = c : c =-3,0,3; z = u : u =0,- Pi /2,- Pi }.



In point 3 we will consider the situation for $\boldsymbol{p(a, z) = \frac{a^2}{2} + P(z)}$**;**

Where $\boldsymbol{P(z)}$ —arbitrary function; the equation takes the form:

$$a_{,xx} + a_{,yy} = -a.$$

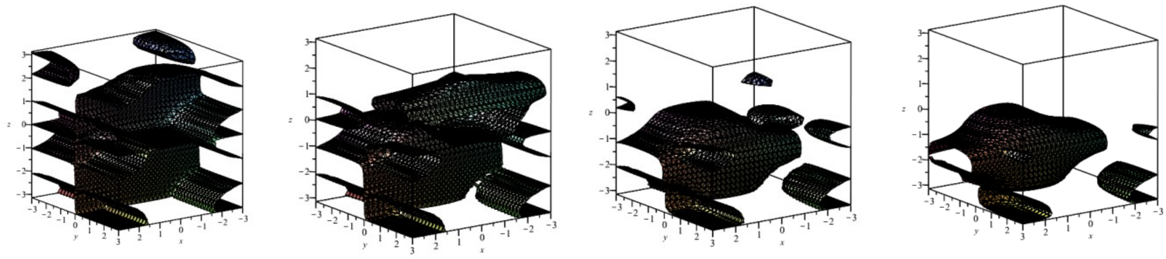
Let us give some specific examples of this situation.

Example 10.

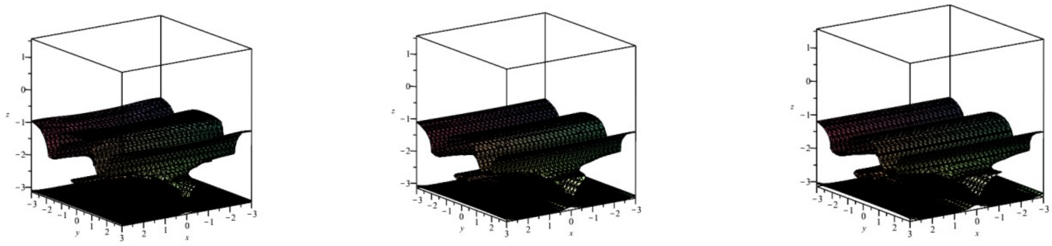
$$a := (x, y, z) \to \exp(-z) * \cos(x * \cos(z)) * \sin(y * \sin(z))\,; a2 := diff(a(x, y, z), x);$$

$$a2 := diff(a(x, y, z), y).$$

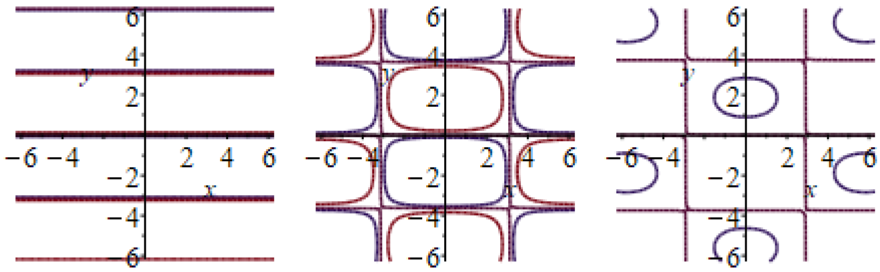
Let's plot graphs for a=c: c=0.01, 0.1, 0.4, 1.



Graphs for the square of the stress H ^{2} = c , c = 3, 5, 7.

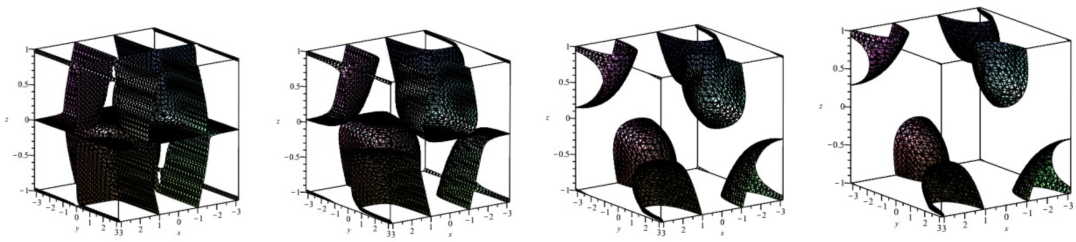


Let us construct graphs of magnetic field lines { a = c : c =-1/2, 0, ¼; z = u : u =- Pi /2,- Pi /3,1).

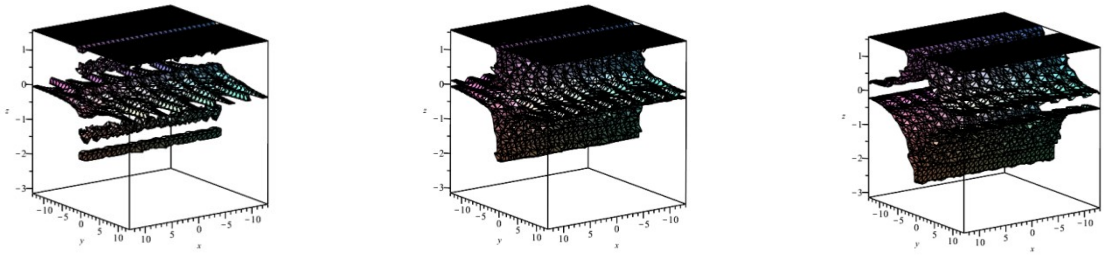


Example 11. Let us choose the potential in the following form

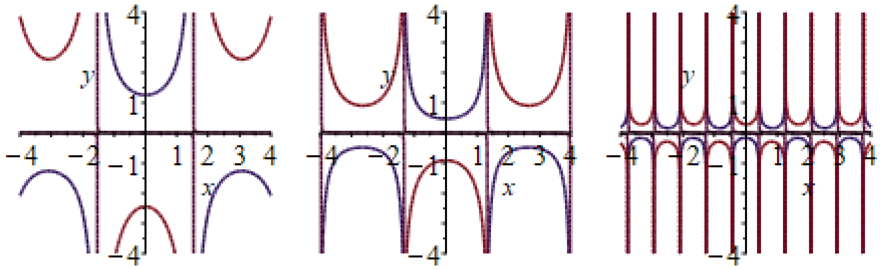
$a := (x.y.z) \ \to \cos(z) * \cos(x * \cosh(z)) * \sinh(y * \sinh(z))$ with parameters a=c: c=0.01, 0.1, 0.5, 1; and plot the graphs.



Let us construct graphs for the square of the magnetic field strength H ^{2} = c , with c = 0.15, 1, 140. .



Let us construct graphs of magnetic field lines { a = c : c =-1/2, 0, 1/4; z = u : u =0.2, 0.6, 2)}.



Conclusion. The conditions of plasma equilibrium in an external potential field are reduced to a single differential equation for the Euler potentials of the magnetic field strength; equilibrium states are constructed in which the equipotential surfaces of the external field are magnetic surfaces.

ЛИТЕРАТУРА

1. Ландау Л.Д., Лифшиц Е.М. Электродинамика сплошных сред. М. Физматлит. 2004.
2. Основы физики плазмы, под ред. А. А. Галеева, Р.Судана, т. 1 - 2, М., 1983.
3. Шаповалов В.Н., Шаповалова О.В. Деп. ВИНИТИ №3318-В99.
4. Янин С.Н. Пекции по основам физики плазмы. Томск. Изд. ТГУ. 2012.
5. Шаповалов В.Н., Шаповалова О.В. Деп. ВИНИТИ №2512-В99. 1999.
6. Шаповалов В.Н., Джимбеева Л.Н. Симметрия и разделение переменных в уравнениях математической физики [Текст]: монография / В.Н. Шаповалов, Л.Н. Джимбеева. – Элиста: Изд-во Калм. ун-та, 2024. – 186 с.
7. Шаповалов В.Н., Джимбеева Л.Н., Сангаджиева Е. В. Об одном классе стационарных состояний идеальной плазмы. //Сборник трудов: Актуальные проблемы математики и физики - Элиста.-/ 2018.
8. Шаповалов В.Н., Джимбеева Л.Н., Субанкул кызы К. Об одном классе стационарных состояний идеальной плазмы // Труды X регион. научно- практич. конф. «АПСФМ». – /Элиста: Изд-во КалмГУ, 2019.
9. Брушлинский К. В. Математические модели плазмы в проектах Морозова / К. В. Брушлинский // Физика плазмы -2019.
10. Морозов А.И., Соловьев Л.С. Стационарные течения плазмы в магнитном поле// Вопросы теории плазмы, вып.8. / Под ред. М.А. Леонтовича. М.: Атомиздат, 1974.
11. Зуева Н.М., Соловьев Л.С., К нелинейной теории газодинамических неустойчивостей / Москва: ИАЭ,- 1980.
12. Шаповалов В.Н., Джимбеева Л.Н., Умкеева Б.В. Некоторые трансляционно-инвариантные конфигурации плазмы. // Труды X регион. научно- практич. конф. «АПСФМ». – Элиста: Изд-во КалмГУ, 2020.

REFERENCES

1. Landau L.D., Lifshits E.M. Ehlektrodinamika sploshnykh sred. M. Fizmatlit. 2004.
2. Osnovy fiziki plazmy, pod red. A. A. Galeeva, R.Sudana, t. 1 - 2, M., 1983.
3. Shapovalov V.N., Shapovalova O.V. Dep. VINITI №3318-V99.
4. Yanin S.N. Pektsii po osnovam fiziki plazmy. Tomsk. Izd. TGU. 2012.
5. Shapovalov V.N., Shapovalova O.V. Dep. VINITI №2512-V99. 1999.
6. Shapovalov V.N., Dzhimbeeva L.N. Simmetriya i razdelenie peremennykh v uravneniyakh matematicheskoi fiziki [Tekst]: monografiya / V.N. Shapovalov, L.N. Dzhimbeeva. – Ehlista: Izd-vo Kalm. un-ta, 2024. – 186 s.
7. Shapovalov V.N., Dzhimbeeva L.N., Sangadzhieva E. V. Ob odnom klasse statsionarnykh sostoyanii ideal'noi plazmy. //Sbornik trudov: Aktual'nye problemy matematiki i fiziki - Ehlista.-/ 2018.
8. Shapovalov V.N., Dzhimbeeva L.N., Subankul kyzy K. Ob odnom klasse statsionarnykh sostoyanii ideal'noi plazmy // Trudy X region. nauchno- praktich. konf. «APSFM». – /Ehlista: Izd-vo KalMGU, 2019.

9. Brushlinskii K. V. Matematicheskie modeli plazmy v proektakh Morozova / K. V. Brushlinskii // Fizika plazmy -2019.

10. Morozov A.I., Solov'ev L.S. Statsionarnye techeniya plazmy v magnitnom pole// Voprosy teorii plazmy, vyp.8. / Pod red. M.A. Leontovicha. M.: Atomizdat, 1974.

11. Zueva H.M., Solov'ev L.S., K nelineinoi teorii gazodinamicheskikh neustoichivostei / Moskva: IAEH,- 1980.

12. Shapovalov V.N., Dzhimbeeva L.N., Umkeeva B.V. Nekotorye translyatsionno-invariantnye konfiguratsii plazmy. // Trudy X region. nauchno- praktich. konf. «APSFM». – Ehlista: Izd-vo KalMGU, 2020.

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Вклад авторов: все авторы внесли равный вклад в подготовку публикации.
Конфликт интересов: авторы заявляют об отсутствии конфликта интересов.
Статья поступила в редакцию 01.08.2025;
одобрена после рецензирования 12.09.2025;
принята к публикации 01.10.2025

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Contribution of the authors: the authors contributed equally to this article.
Conflict of interest: the authors declare no conflicts of interests.

The article was submitted: 01.08.2025;
approved after reviewing:12.09.2025;
accepted for publication:01.10.2025.