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Разработка концептуального подхода к построению математической модели оптимальной сигнальной последовательности

Development of a conceptual approach to the construction of a mathematical model of the optimal signal sequence

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Аннотация. *Разработка и построение математических моделей систем дискретных сигналов, базирующихся на информационном задании сигнальных последовательностей, требований к ним и построении операторных семейств, открывает дополнительные возможности в решении ряда задач, труднореализуемых для методов моделирования, разработанных ранее. В статье предложен подход к построению математической модели оптимальной сигнальной последовательности, основанный на поэтапном синтезе, учитывающем специфику построения пространства управлений и математического регуляризатора, а также структуру операторов контроля и выбора терминального управления. Обоснованы и сформулированы шесть этапов построения математической модели, которая может лежать в основе алгоритма при синтезе оптимальных сигнальных последовательностей.*

Ключевые слова: сигнальная последовательность, пространство управлений, пространство характеристик, математический регуляризатор, оператор контроля, моделирование сигнальных конструкций

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Abstract. *The development and construction of mathematical models of discrete signal systems based on the information specification of signal sequences, requirements for them and the construction of operator families opens up additional opportunities for solving a number of problems that are difficult to implement for modeling methods developed earlier. The article proposes an approach to constructing a mathematical model of the optimal signal sequence, based on a step-by-step synthesis, taking into account the specifics of constructing the control space and mathematical regularizer, as well as the structure of control operators and the choice of terminal control. Substantiated and formulated six stages of constructing a mathematical model that can underlie the algorithm in the synthesis of optimal signal sequences.*

Keywords: signal sequence, control space, feature space, mathematical regularizer, control operator, simulation of signal structures

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Introduction. An analysis of a number of works [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] shows that the use of complex broadband signals makes it possible to increase the noise immunity of a communication system. However, the use of these signals is focused on the presence of channels with an excess of frequency resources, which is only possible for channels with large frequency capacity, and noise-resistant coding is accompanied by a significant reduction in the information transmission rate, which is unacceptable in conditions of transmitting large amounts of data.

Materials and research methods. The works [12, 14, 15, 16, 17, 18] laid the theoretical foundations for optimizing information systems and proved the possibility of creating synchronous systems with noise immunity and information transfer rates approaching the maximum possible, based on the use of optimal signal sequences. In this case, the structure of the signals and methods of their synthesis are not considered, since the problem is solved only for a channel with additive white Gaussian noise without taking into account restrictions on the spectrum width and peak factor of the signal sequence, subject to ideal synchronization. In turn, real radio channels differ very significantly from the channels considered in the above sources, both in the limited potential capabilities of the frequency and dynamic ranges, and in the interference environment, characterized by the presence of additional powerful narrow-band, concentrated, pulsed and multiplicative interference of natural and intentional origin.

Taking these factors into account places a number of very significant requirements on the characteristics of signals, the implementation of which can be ensured by an appropriate choice of their structure.

The purpose of the article is to substantiate and develop an approach to constructing a mathematical model of the optimal signal sequence, taking into account the specifics of constructing the control space and mathematical regularizer, as well as the structure of control and terminal control operators.

Research results and their discussion. Conceptual approach to constructing a mathematical model of the optimal signal sequence. Using general approaches to mathematical modeling of signal structures, any signal sequence can be described by a set of functions [18]:

$$\vec{S} = \left\{ S_1(t); S_2(t); ...; S_n(t) \right\}, \tag{1}$$

where $S_n(t) \in L[0;t]$ for n = 1, 2, ..., and $\left\{\vec{S}\right\} = S$ is a linear space.

During modeling, it is advisable to use only functions with finite energy on a finite interval as signals:

$$\int_0^T S_n^2(t)\, dt < \infty . \tag{2}$$

If the transmission and processing of signals is carried out using linear algorithms, then the functions used to describe such signals must be linearly independent, that is, when developing a mathematical model, it is enough to limit ourselves to enumerating possible space bases $L2\,[0\,;T\,]$.

To assess the effectiveness of signal sequences, it is necessary to analyze the space of characteristics $Y = \left[\vec{Y}\right]$, the spectral and correlation characteristics of the simulated signals can be used as components of this space.

Given a limited number of signal characteristics, we can define Y as an arithmetic space whose dimension is limited by the number of characteristics used.

Comparing $\vec{S}$ and $\vec{Y}$ it can be argued that by assigning a control operator to each system $\vec{S}$ a supersystem can be assigned $\vec{Y}$ [18]:

$$C : S \rightarrow Y . \tag{3}$$

If $\vec{Y}_0$ it represents a set of characteristics that satisfy a number of requirements for signal sequences, then, taking into account the requirements for the transmission system, $Y_0 = \left\{ \vec{Y}_0 \right\}$ it forms a subset of the space *Y* that satisfies the given requirements [19].

If the characteristics are within the specified limits, then Y_0 will be a parallelepiped:

$$Y_0 = \left\{ \vec{Y} \,\middle|\, a_i \le Y_i \le b_i \right\}. \tag{4}$$

Given the optimal requirements $\vec{y}_0$ and permissible deviation, Y_0 will be a ball:

$$Y_0 = \left\{ \vec{Y} \,\middle|\, \left\| Y - Y_0 \right\| < e \right\}. \tag{5}$$

Also, Y_0 can be a finite set consisting of individual points:

$$Y_0 = \left\{ \vec{Y}_1; \vec{Y}_2; \ldots; \vec{Y}_n \right\}. \tag{6}$$

By choosing $\vec{S}_0$ such that [19]

$$C\left(\vec{S}_0\right) = \vec{Y}_0, \tag{7}$$

the description of the simulated signal sequence that satisfies the requirements is exhausted.

Considering that the task of constructing a mathematical model involves solving the operator equation [19]:

$$C\left(\vec{S}\right) = \vec{Y};\ \vec{Y} \in Y_0, \tag{8}$$

then it can be considered as an optimal design problem. In this case, it is necessary to introduce the concepts of a task for a project and a project, defining them as carriers of information at the points $\vec{Y} \in Y$ and $\vec{S} \in S$.

The project assignment will be a set of individual requirements ζ for vector characteristics $\vec{Y}$ [19]:

$$З\left(\vec{Y}\right) = \left\{ \zeta \right\}. \tag{9}$$

In general, a project assignment may include unequal requirements [19], which can be ordered by a preference relationship. The task obtained in this way is not an intersection of requirements, but can be reduced to this when equivalent requirements are presented.

If the project assignment is presented as a system of nested requirements, ordered by preference by importance, then it becomes possible to eliminate the empty set.

A project should be understood as a set of information π about the signal sequence model $\vec{S}$ [18]:

$$\Pi\left(\vec{S}\right) = \left\{ \pi \right\}. \tag{10}$$

Thus, the task of constructing a mathematical model of a signal sequence that satisfies a set of requirements can be formulated as follows:

given: $S, Y, C : S \to Y,\ З\left(\vec{Y}\right) = \left\{ \zeta \right\}$;

find: $\Pi\left(\vec{S}\right) = \left\{ \pi \right\}$.

In the general case, the main stages of constructing a mathematical model of a signal sequence that satisfies specified requirements can be formulated as follows:

1. Construction of model space. At this stage, issues of constructing a mathematical model that adequately reflects the optimal signal sequence based on the principle of micro- and macro-complexity are considered.
2. Construction of the space of characteristics. At this stage, the requirements for optimal signal sequences for the initial data are formulated and the corresponding characteristics are selected, on the basis of which the space of characteristics *Y is constructed.*
3. Construction of the control operator. This stage includes the construction of an operator that evaluates the characteristics of the analyzed signal sequences in the process of solving the problem of constructing a mathematical model of the optimal signal sequence. The

control operator assigns each coordinate an $\vec{S}$ element of the space of characteristics *Y*. This follows from the fact that

$$\left\{0;\ ...;\ S_n\left(t\right);\ ...\right\} \in S. \tag{11}$$

When evaluating a simulated signal sequence, it must be taken into account that the construction of a control operator is associated with a large number of calculations, since during the evaluation process it is necessary to select extreme values of the corresponding characteristics for the coordinates of signal points.

4. Defining a regularizer for the control operator. At this stage, the problem of constructing a regularizer *R* and a control space *U is solved.* Selecting a compact set in *S* that limits the spread of models of ensembles of signals with equivalent characteristics is the main role of the regularizer.

5. Constructing a terminal control selection operator. This stage includes establishing and deducing the dependence of the control space element on the characteristics of the signals

$$\vec{S} = \vec{S}\left(Y_1; Y_2; ...; Y_n\right). \tag{12}$$

After which a set U_k is determined such that

$$CR\left(U_k\right) \in \zeta_k. \tag{13}$$

6. Building project information. At this stage, using a regularizer for the established model parameters (selected elements of the control space), a mathematical model of the optimal signal sequence that meets the specified requirements is developed:

$$\pi_k = R\left(U_k\right). \tag{14}$$

The construction of $\left\{\pi_k\right\} = \Pi\left(\vec{S}\right)$ the problem of developing a mathematical model of the optimal signal sequence is completed.

Conclusion. Thus, the basis for constructing a mathematical model of the optimal signal sequence will be the six stages formulated above, while the specifics of the control space and the mathematical regularizer, as well as the structure of control operators and selection of terminal control, must be taken into account. The proposed approach makes it possible to develop systems of discrete signals that meet the requirements for them, as well as to simplify the implementation of the algorithm for their synthesis by formalizing each stage.

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