Современная наука и инновации.
2023. № 3 (43). С. 19-24
Modern Science and Innovations.
2023;3(43):19-24

ТЕХНИЧЕСКИЕ НАУКИ / TECHNICAL SCIENCES

ИНФОРМАТИКА, ВЫЧИСЛИТЕЛЬНАЯ ТЕХНИКА И УПРАВЛЕНИЕ / INFORMATICS, COMPUTER ENGINEERING AND MANAGEMENT

Научная статья / Original article

УДК 621.391
https://doi.org/10.37493/2307-910X.2023.3.2

Владимир Михайлович Самус
[Vladimir M. Samus]

Использование ортогональных преобразований для получения сигнальных последовательностей с новыми автокорреляционными и спектральными характеристиками

Using orthogonal transformations to obtain signal sequences with new autocorrelation and spectral characteristics

Северо-Кавказский федеральный университет, г. Ставрополь, Россия, vladimir-samus@mail.ru / North-Caucasus Federal University, Stavropol, Russia, vladimir-samus@mail.ru

Аннотация. *В статье разработан способ получения последовательностей дискретных сигналов, значительно упрощающий процесс получения сигнальных последовательностей с новыми автокорреляционными и спектральными характеристиками, основанный на свойствах ортогональных преобразований базисных функций, который включает в себя три этапа: построение матрицы перехода; расчет координат сигнальных точек в новом базисе; представление системы сигналов в базисе смещенных единичных импульсов с использованием координат в новом базисе.*

Ключевые слова: коэффициент взаимной корреляции, ортогональный базис, обратная матрица, матрица перехода, базис смещённых единичных импульсов

Для цитирования: *Самус В. М. Использование ортогональных преобразований для получения сигнальных последовательностей с новыми автокорреляционными и спектральными характеристиками // Современная наука и инновации. 2023. № 3 (43). С. 19-24.* https://doi.org/10.37493/2307-910X.2023.3.2

Abstract. *The paper developed a method for obtaining sequences of discrete signals, which greatly simplifies the process of obtaining signal sequences with new autocorrelation and spectral characteristics, based on the properties of orthogonal transformations of basis functions, which includes three stages: building a transition matrix; calculation of coordinates of signal points in a new basis; representation of the signal system in the basis of displaced unit pulses using coordinates in the new basis.*

Keywords: cross-correlation coefficient, orthogonal basis, inverse matrix, transition matrix, shifted unit pulse basis

For citation: *Samus VM. Using orthogonal transformations to obtain signal sequences with new autocorrelation and spectral characteristics. Modern Science and Innovations. 2023;3(43):19-24.* https://doi.org/10.37493/2307-910X.2023.3.2

Introduction. The use of complex broadband signals in combination with optimal reception methods makes it possible to increase the noise immunity of the communication system as a whole, which is confirmed by publications [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

One of the problems of noise-resistant coding is the problem of optimal placement of signal points in n -dimensional space. Optimal placement implies uniform distribution of signal points at the maximum possible distance from each other. In multidimensional discrete geometry, such problems are reduced to the problem of densest packing of identical balls in n -dimensional space [13], provided that the centers of the balls are signal points.

As a rule, the problem of dense packing has many solutions, which, however, are not widely used in practice due to their inherent disadvantages, which primarily include the limited scope of the bases covered [15].

Materials and research methods. The purpose of the article is to develop a method for obtaining sequences of discrete signals, which will significantly simplify the process of obtaining sequences of signals with new characteristics and significantly increase their number.

Research results and their discussion.

Obtaining signal sequences with new autocorrelation and spectral characteristics based on orthogonal transformations

Any signal-code sequence can be represented as a linear combination of basis vectors:

$$\begin{aligned} &\vec{S}_1 = a_{11}\vec{\Theta}_1 + a_{12}\vec{\Theta}_2 + a_{13}\vec{\Theta}_3 + \ldots + a_{1n}\vec{\Theta}_n; \\ &\vec{S}_2 = a_{21}\vec{\Theta}_1 + a_{22}\vec{\Theta}_2 + a_{23}\vec{\Theta}_3 + \ldots + a_{2n}\vec{\Theta}_n; \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\vec{S}_m = a_{m1}\vec{\Theta}_1 + a_{m2}\vec{\Theta}_2 + a_{m3}\vec{\Theta}_3 + \ldots + a_{mn}\vec{\Theta}_n, \end{aligned} \tag{1}$$

where $\{\vec{\Theta}\}$ is a sequence *of n* basis functions.

The proposed method is based on the orthogonal transformation of the coordinates of signal points when transferring them from the original basis to a new one, followed by representation of the coordinates of the signal points in the basis of time-shifted unit pulses

$$\vec{\eta}_k(t) = rect(t - k\Delta t), \tag{2}$$

Moreover, if the old and new bases are orthonormal, then the lengths of the vectors and the angles between them do not change. Consequently, the cross-correlation coefficient does not change. And the spectral and energy characteristics, as well as the autocorrelation function, change due to a change in the structure of the signal, determined by the coordinates in the new basis.

For two bases chosen in n -dimensional space, there is an operator T , which transforms a basis $\{\vec{\Theta}\}$ into a basis $\{\vec{\Theta}^*\}$ [16, 17, 18, 19]

$$\vec{\Theta}^* = \tilde{T} \cdot \vec{\Theta}. \tag{3}$$

The operator T corresponds to a non-singular matrix

$$T = \|t_{ij}\| = \begin{pmatrix} t_{11} & t_{12} & \ldots & t_{1n} \\ t_{21} & t_{22} & \ldots & t_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ t_{n1} & t_{n2} & \ldots & t_{nn} \end{pmatrix}, \tag{4}$$

the columns of which are the coefficients of expansions of new coordinate vectors into old basic ones [20, 21]

$$\begin{aligned} &\vec{\Theta}_1^* = t_{11}\vec{\Theta}_1 + t_{21}\vec{\Theta}_2 + \ldots + t_{n1}\vec{\Theta}_n; \\ &\vec{\Theta}_2^* = t_{12}\vec{\Theta}_1 + t_{22}\vec{\Theta}_2 + \ldots + t_{n2}\vec{\Theta}_n; \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\vec{\Theta}_n^* = t_{1n}\vec{\Theta}_1 + t_{2n}\vec{\Theta}_2 + \ldots + t_{nn}\vec{\Theta}_n. \end{aligned} \tag{5}$$

The sequence of signals in the new basis can be represented:

$$\vec{S}_1 = a_{11}^*\vec{\Theta}_1^* + a_{12}^*\vec{\Theta}_2^* + a_{13}^*\vec{\Theta}_3^* + \ldots + a_{1n}^*\vec{\Theta}_n^*;$$
$$\vec{S}_2 = a_{21}^*\vec{\Theta}_1^* + a_{22}^*\vec{\Theta}_2^* + a_{23}^*\vec{\Theta}_3^* + \ldots + a_{2n}^*\vec{\Theta}_n^*; \qquad (6)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\vec{S}_m = a_{m1}^*\vec{\Theta}_1^* + a_{m2}^*\vec{\Theta}_2^* + a_{m3}^*\vec{\Theta}_3^* + \ldots + a_{mn}^*\vec{\Theta}_n^*,$$

where $\{a_{ij}^*\}$ are the coordinates of the signal points in the new basis.

Provided that the signal sequence decomposition is unique in the old basis:

$$a_{11} = a_{11}^* t_{11} + a_{12}^* t_{12} + \ldots + a_{1n}^* t_{1n};$$
$$a_{12} = a_{11}^* t_{21} + a_{12}^* t_{22} + \ldots + a_{1n}^* t_{2n}; \qquad (7)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{1n} = a_{11}^* t_{n1} + a_{12}^* t_{n2} + \ldots + a_{1n}^* t_{nn}.$$

Taking into account (4), system (7) can be rewritten in matrix form

$$A = A^* \cdot T \,, \qquad (8)$$

Where

$$A = \begin{pmatrix} a_{11} \\ a_{12} \\ \ldots \\ a_{1n} \end{pmatrix}; \quad A^* = \begin{pmatrix} a_{11}^* \\ a_{12}^* \\ \ldots \\ a_{1n}^* \end{pmatrix}.$$

It is easy to show that relation (8) remains valid for other signal sequences (1). In this case, the relationship that determines the new coordinates through the old ones, provided that matrix (4) is non-singular, has the form

$$A^* = A \cdot T^{-1}, \qquad (9)$$

where T^{-1} is the inverse matrix defined by the equality [6, 7]

$$T^{-1} = \frac{1}{\det T} T_{ij}^T = \begin{pmatrix} t_{11} & t_{21} & \ldots & t_{n1} \\ t_{12} & t_{22} & \ldots & t_{n2} \\ \ldots & \ldots & \ldots & \ldots \\ t_{1n} & t_{2n} & \ldots & t_{nn} \end{pmatrix}, \qquad (10)$$

where T_{ij}^T is a transposed matrix T_{ij} composed of algebraic complements of elements t *ij* *of* matrix *T*.

With known coordinates in the original basis and the transition matrix defined by expression (10), using relation (9) it is possible to determine the coordinates of each vector signal in the new basis.

The next step is to present the new coordinates obtained using relation (9) in the basis of shifted unit pulses (2), and a new sequence of signals will be obtained:

$$\vec{S}_1^* = a_{11}^*\vec{\eta}_1 + a_{12}^*\vec{\eta}_2 + a_{13}^*\vec{\eta}_3 + \ldots + a_{1n}^*\vec{\eta}_n;$$
$$\vec{S}_2^* = a_{21}^*\vec{\eta}_1 + a_{22}^*\vec{\eta}_2 + a_{23}^*\vec{\eta}_3 + \ldots + a_{2n}^*\vec{\eta}_n;$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\vec{S}_m^* = a_{m1}^*\vec{\eta}_1 + a_{m2}^*\vec{\eta}_2 + a_{m3}^*\vec{\eta}_3 + \ldots + a_{mn}^*\vec{\eta}_n.$$

(eleven)

The proof of the above statements is based on a well-known statement in linear algebra [16, 17]: an orthogonal transformation of coordinates does not change the scalar product of vectors.

This statement has consequences:

Corollary 1. An orthogonal transformation does not change the lengths of vectors and the angles between vectors.

Corollary 2. An orthogonal transformation transforms an orthonormal basis into an orthonormal one.

Corollary 3. An orthogonal transformation does not change the distance between vectors (based on Corollary 2).

Based on the above statement and corollaries from it, we can conclude that the relative position of signal vectors during the transition from one orthonormal basis to another does not change while the spectral and autocorrelation characteristics of the signals change simultaneously due to a change in their structure, determined by the coordinates in the new basis.

Conclusion. From the above it follows that if an orthogonal transformation based on the matrix *T is applied to the existing signal sequence of discrete signals presented in the original basis*, and then the resulting coordinates are presented in the basis of time-shifted unit pulses, then a new sequence of signals will be obtained in the new basis. In this case, the coordinates of the signal points are determined by formula (9) using known coordinates (1) and the inverse matrix T^{-1} specified by relation (10). Matrix T is determined from system (5) based on a comparison of the original basis { $\vec{\Theta}$ } and the new { $\vec{\Theta}^*$ } by representing the coordinates of the basis vectors $\vec{\Theta}_k$ in the basis { $\vec{\Theta}^*$ }. Taking into account the existing apparatus for synthesizing discrete orthogonal bases [15], it is possible to obtain an unlimited number of signal sequences with new spectral and correlation characteristics, including those that satisfy specified requirements.

ЛИТЕРАТУРА

1. Варакин Л. Е. Системы связи с шумоподобными сигналами. М.: Радио и связь, 1985. 384 с.

2. Диксон Р. К. Широкополосные системы. Пер. с англ. / под ред. В. И. Журавлева. М.: Связь, 1979. 304 с.

3. Дядюнов Н. Г., Сенин А. И. Ортогональные и квазиортогональные сигналы. М.: Связь, 1977. 222 с.

4. Передача цифровой информации. Пер. с англ. / под ред. С. И. Самойленко. М.: Издательство иностранной литературы, 1963. 325 с.

5. Зайдлер Е. Системы передачи дискретной информации. Пер. с польск. / под ред. Б. Р. Левина. 7-й вып. М.: Связь, 1977. 512 с.

6. Пенин П. И., Филиппов Л. И. Радиотехнические системы передачи информации. М.: Радио и связь, 1984. 256 с.

7. Системы подвижной радиосвязи / под ред. И. М. Пышкина / И. М. Пышкин, И. И. Дежурный, В. Н. Талызин, Г. Д. Чвилев. М.: Радио и связь, 1986. 328 с.

8. Хармут Х. Ф. Передача информации ортогональными функциями. Пер. с англ. Н. Г. Дядюнова, А. И. Сенина. М.: Связь, 1975. 267 с.

9. Балакришнан А. В. Теория связи. М.: Связь, 1972. 231 с.

10. Семенов А. М., Сикарев А. А. Широкополосная радиосвязь. М.: МО СССР, 1970. 278 с.

11. Варакин Л. Е. Теория сложных сигналов. М.: Советское радио, 1978. 199 с.

12. Пестряков В. Б., Афанасьев В. П., Гурвиц В. Л. Шумоподобные сигналы в системах передачи информации / под ред. В. Б. Пестрякова. М.: Советское радио, 1973. 424 с.

13. Тот Л. Ф. Расположение на плоскости, на сфере и в пространстве. Пер. с нем. Н. М. Макаровой. М.: Физматлит, 1958. 363 с.

14. Allen RL, Mills DW. Signal analysis. Time, frequency, scale, and structure. Wiley-Interscience, 2004. 382 p.

15. Попенко В. С. Векторный синтез ансамблей ортогональных сигналов. Ч. 3. Ставрополь: МО РФ, 1993. 150 с.

16. Блох Э. Л., Лошинский Л. И., Турин В. Я. Основы линейной алгебры и некоторые её приложения. М.: Высшая школа, 1971. 256 с.

17. Головина Л. И. Линейная алгебра и некоторые её приложения. М.: Наука, 1971. 288 с.

18. Мальцев А. И. Основы линейной алгебры. М.: Наука, 1970. 255 с.

19. Wang R. Introduction to Orthogonal Transforms: With Applications in Data Processing and Analysis. Cambridge University Press, 2012. 590 p.

20. Боревич З. И. Определители и матрицы. М.: Наука, 1970. 540 с.

21. Маркус М., Минк Х. Обзор по теории матриц и матричных неравенств. Пер. с англ. / под ред. В. Б. Лидского. М.: Наука, 1972. 232 с.

REFERENCES

1. Varakin LE. Sistemy svyazi s shumopodobnymi signalami. M.: Radio i svyaz', 1985. 384 p.

2. Dikson RK. Shirokopolosnye sistemy. Per. s angl. Pod red. VI. Zhuravleva. M.: Svyaz', 1979. 304 p.

3. Dyadyunov NG, Senin AI. Ortogonal'nye i kvaziortogonal'nye signaly. M.: Svyaz', 1977. 222 p.

4. Peredacha tsifrovoi informatsii. Per. s angl. Pod red. S. I. Samoilenko. M.: Izdatel'stvo inostrannoi literatury, 1963. 325 p.

5. Zaidler E. Sistemy peredachi diskretnoi informatsii. Per. s pol'sk. Pod red. B. R. Levina. 7-i vyp. M.: Svyaz', 1977. 512 p.

6. Penin PI, Filippov LI. Radiotekhnicheskie sistemy peredachi informatsii. M.: Radio i svyaz', 1984. 256 p.

7. Sistemy podvizhnoi radiosvyazi. Pod red. IM. Pyshkina / IM. Pyshkin, II. Dezhurnyi, VN. Talyzin, GD. Chvilev. M.: Radio i svyaz', 1986. 328 p.

8. Kharmut KhF. Peredacha informatsii ortogonal'nymi funktsiyami. Per. s angl. N. G. Dyadyunova, A. I. Senina. M.: Svyaz', 1975. 267 p.

9. Balakrishnan AV. Teoriya svyazi. M.: Svyaz', 1972. 231 p.

10. Semenov AM, Sikarev AA. Shirokopolosnaya radiosvyaz'. M.: MO SSSR, 1970. 278 p.

11. Varakin LE. Teoriya slozhnykh signalov. M.: Sovetskoe radio, 1978. 199 p.

12. Pestryakov VB, Afanasev VP, Gurvits VL. Shumopodobnye signaly v sistemakh peredachi informatsii / pod red. V. B. Pestryakova. M.: Sovetskoe radio, 1973. 424 p.

13. Tot LF. Raspolozhenie na ploskosti, na sfere i v prostranstve. Per. s nem. N. M. Makarovoi. M.: Fizmatlit, 1958. 363 p.

14. Allen RL, Mills DW. Signal analysis. Time, frequency, scale, and structure. Wiley-Interscience, 2004. 382 p.

15. Popenko VS. Vektornyi sintez ansamblei ortogonal'nykh signalov. CH. 3. Stavropol: MO RF, 1993. 150 p.

16. Blokh EhL, Loshinskii LI., Turin VYa. Osnovy lineinoi algebry i nekotorye ee prilozheniya. M.: Vysshaya shkola, 1971. 256 p.

17. Golovina LI. Lineinaya algebra i nekotorye ee prilozheniya. M.: Nauka, 1971. 288 p.

18. Mal'tsev AI. Osnovy lineinoi algebry. M.: Nauka, 1970. 255 p.

19. Wang R. Introduction to Orthogonal Transforms: With Applications in Data Processing and Analysis. Cambridge University Press, 2012. 590 p.

20. Borevich ZI. Opredeliteli i matritsy. M.: Nauka, 1970. 540 p.

21. Markus M, Mink Kh. Obzor po teorii matrits i matrichnykh neravenstv. Per. s angl. / pod red. VB. Lidskogo. M.: Nauka, 1972. 232 p.

ИНФОРМАЦИЯ ОБ АВТОРЕ

Владимир Михайлович Самус – аспирант кафедры информационной безопасности автоматизированных систем, Северо-Кавказский федеральный университет, пр. Кулакова, 2 (корпус 9), г. Ставрополь, 355029, Россия, +79654600393

INFORMATION ABOUT THE AUTHOR

Vladimir M. Samus – Doctoral Student, the Postgraduate at the Department of Information Security of Automated Systems, North-Caucasian Federal University, Kulakova Ave, 2 (building 9), Stavropol, 355029, Russia, +79654600393

Конфликт интересов: автор заявляет об отсутствии конфликта интересов.
Conflict of interest: the author declares no conflicts of interests.

Статья поступила в редакцию: 13.07.2023;
одобрена после рецензирования: 16.08.2023;
принята к публикации: 06.09.2023.

The article was submitted: 13.07.2023;
approved after reviewing: 16.08.2023;
accepted for publication: 06.09.2023.